

The Effect of the Hedge Horizon on Optimal Hedge Size and Effectiveness when Prices are Cointegrated

July 2010

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Abstract

This study compares three alternative regression specifications for sizing hedge positions and measuring hedge effectiveness, to inform risk management decisions and efforts to comply with hedge accounting rules: a simple regression on price levels, a simple regression on price changes and an error correction model (ECM). We show that, when the prices of the hedged item and the hedging instrument are cointegrated, all three specifications provide useful information to the risk manager. Furthermore, the latter two specifications yield similar results which depend on the hedge horizon (i.e., the time frame for measuring price changes). In particular, the estimated hedge ratio and regression R^2 will both be small when price changes are measured over short intervals, but as the hedge horizon is lengthened both measures will increase toward one. These results imply that, when prices are cointegrated, a longer hedge horizon will yield an optimal hedge ratio closer to one, while enhancing hedge effectiveness and the ability to comply with hedge accounting rules.

Key Words: risk management; hedge ratio; hedge effectiveness; hedge accounting; cointegration.
JEL Classification: G18, G38

*Corresponding author. We thank Henk Berkman, Audra Boone, Jeff Mercer, Shane Moriarity, Panos Patatoukas, Peter Wells, Jide Wintoki, and seminar participants at the American Accounting Association Conference, the Financial Management Association Conference, and the Southern Finance Association Conference.

Risk managers conduct statistical analysis to size hedge positions (i.e., estimate optimal hedge ratios) and measure anticipated hedge effectiveness, in order to guide their risk management activities. Hedge accounting rules reinforce this behavior, by requiring hedgers to validate their expectation that a prospective derivatives hedge will be effective in offsetting a particular exposure. The standard textbook approach to measure anticipated hedge effectiveness is the R^2 from a simple regression on price changes in the hedged item and the hedging instrument, where price changes are measured over the hedge horizon and the slope coefficient is the optimal hedge ratio. A necessary condition to qualify for special hedge accounting treatment is an $R^2 \geq .80$.

This study compares the standard textbook approach with two alternative specifications: a simple regression on price levels and an error correction model (ECM) on price changes. In theory, the proper specification depends on the time series properties of the price series involved. We are interested in how these three specifications behave as we extend the hedge horizon, when prices are cointegrated. It is critical for hedgers to understand this behavior, so they can make informed decisions as they strive to manage risk while complying with hedge accounting rules.

We analyze the relation between the hedge horizon and the hedge ratio and regression R^2 , when the prices of the hedged item and hedging instrument are cointegrated. For a prototypical ECM, we prove that the estimated hedge ratio and regression R^2 will both be small for short horizons. However, as the hedge horizon is extended, the ECM will generate an error correction coefficient that converges toward negative one, while the hedge ratio and R^2 increase toward positive one. Furthermore, the latter two results also occur with the standard textbook approach, when the user omits the error correction term and estimates a simple regression on price changes.

We empirically explore these issues with a case study that analyzes optimal hedge size and anticipated hedge effectiveness for a firm using gasoline futures to hedge delivery of gasoline at

six locations. We find the daily gasoline cash and futures prices are cointegrated, consistent with the prototypical ECM. The simple regression on price *levels* is analogous to one component of the cointegrating system, and it yields a slope coefficient and R^2 that both remain close to one as we vary the time frame for measuring price levels from daily to weekly, monthly, quarterly, and six-month intervals. In contrast, the analogous hedge ratio and R^2 from the simple regression or ECM on *daily price changes* are close to zero, but they increase toward one as the hedge horizon is lengthened. Furthermore, the error correction coefficient tends toward negative one as the hedge horizon increases. Importantly, the hedge ratio and R^2 from the textbook approach are nearly identical to those from the ECM, for all time frames examined. This evidence supports our analytical findings and suggests that, if the hedging relation follows the prototypical ECM, then it does not matter if the hedger applies an ECM or the textbook approach.

These analytical and empirical results have profound implications for the use of, and accounting for, derivatives in hedging. First, if the cash and futures prices follow a stable relation consistent with the prototypical ECM, then it is reasonable to anticipate that the optimal hedge will ultimately be effective in offsetting the exposure, provided that the hedger can maintain the positions for a sufficiently long hedge horizon. Second, if data on the cointegrated price series are available for an adequate sample period, then the hedger can be confident in meeting the criterion to qualify for hedge accounting (i.e., obtaining an R^2 close to one), again, if the hedger is willing to consider a long enough hedge horizon. Third, it does not matter if the user estimates an ECM or applies the textbook approach, since both specifications will yield an estimated hedge ratio and R^2 that approach one as we lengthen the hedge horizon.

This article proceeds as follows. Section 1 provides a brief literature review and motivation for our study. Section 2 discusses the relevant accounting and econometric issues.

Section 3 describes accounting rules for a cross-hedge, and alternative regression specifications using price levels or price changes. Section 4 discusses the choice of a hedge horizon, and the implications of cointegration for the regression specification. Section 5 presents our analytical findings, and section 6 provides the case study. A final section concludes.

1. Background and Motivation

A large body of research addresses how the hedging activities of risk managers can enhance firm value.¹ Risk managers need to assess the proper sizes and anticipated effectiveness of their hedge positions, in order to make informed decisions. Accounting rules under Topic 815 (formerly Financial Accounting Standard No. 133) support this need, by requiring hedgers to statistically validate their expectation that a prospective hedge will be “highly” effective in offsetting a particular risk exposure, in order to qualify for special hedge accounting treatment. The common metric for validating this expectation is the R^2 from a regression analysis, while the regression slope coefficient provides an estimate of the optimal hedge ratio. Furthermore, it is understood that a necessary condition to qualify for hedge accounting treatment is a regression $R^2 \geq .80$. However, the proper design of the regression model is a matter of some dispute.²

There is a lively debate on how these derivatives accounting rules affect the risk management activities of firms. Advocates argue that fair value-based recognition (i.e., marking to market through earnings) makes a firm’s use of derivatives more transparent, thereby encouraging prudent risk management and enhancing market efficiency.³ Opponents argue that firms mainly use derivatives to hedge, and fair value disclosure may fail to reflect the gains or losses associated with the exposure being hedged, leading to greater short term income volatility.

¹ For examples of recent work, see Allayannis and Weston (2001), Carter et al. (2006), Jin and Jorion (2006), and Mackay and Moeller (2007).

² The Financial Accounting Standards Board (FASB) has recently released an exposure draft that, if enacted, would lower the requirement from “highly” effective to “reasonably” effective. It is not clear whether or how this change would affect the current practice of requiring the regression R^2 statistic to be at least .80.

³ For example, see Hodder et al. (2006), Kanodia et al. (2000), and Zhang (2009).

Furthermore, opponents emphasize that the requirement for statistical analysis to validate anticipated hedge effectiveness is difficult for practitioners to understand and costly to implement. As a result, this requirement may deter the legitimate use of derivatives for hedging.⁴

The textbook prescription for measuring the optimal hedge ratio and hedge effectiveness is a simple regression on price changes, where the dependent variable relates to the risk exposure (e.g., a cash market price) and the independent variable relates to the hedging instrument (e.g., a futures contract price), while the time frame for measuring price changes matches the hedge horizon (Ederington, 1979; Hull, 2008). Another common industry practice is to estimate a simple regression on price levels rather than price changes.⁵ Finally, a growing body of work recommends an error correction model (ECM) to account for cointegration.⁶

A substantial body of prior work has shown that regression analysis on price changes taken over short time intervals often leads to a small hedge ratio and low R^2 . However, as the hedge horizon is lengthened, the estimated hedge ratio and R^2 measure both tend to increase.⁷ Such behavior presents an obvious problem for a risk manager who might prefer a shorter hedge horizon, but must weigh the consequences for hedge effectiveness and compliance with FAS 133. A common explanation for this behavior is that short run noise in the market tends to cancel out over longer horizons, as the true long run relation is revealed. This explanation, however, lacks a robust theoretical foundation and sheds little light on the underlying causes.

To better understand this behavior, two prior studies model the relation between the hedge horizon and the optimal hedge ratio or measure of hedge effectiveness. Howard and D'Antonio

⁴ For example, see Charnes et al. (2003), DeMarzo and Duffie (1995), Freeman and Wells (2009), Kawaller and Koch (2000), Sapra (2002), and Wong (2000).

⁵ Levy and Sarnat (1984) and Marmer (1986) discuss various drawbacks to the textbook approach. Hill and Schneeweis (1981, 1982) and Dale (1981) debate use of a simple regression on price levels versus price changes.

⁶ For example, see Benz and Hengelbrock (2009), Chou et al. (1996), Ghosh (1993a,b), Kenourgios et al. (2008), Kroner and Sultan (1993), Lien (1996, 2004), and Lien and Tse (1999).

⁷ See Benet (1992), Chou et al. (1996), Ederington (1979), Geppert (1995), Howard and D'Antonio (1991), Hill and Schneeweis (1981, 1982), Lee et al. (2009), Lien (2000), Lien and Luo (1993), and Malliaris and Urrutia (1991a,b).

(1991) build a model in which the optimal hedge ratio depends on the investment horizon. Their results are driven by an assumption of autocorrelation in the spot asset return. Geppert (1995) analyzes the relation between the hedge horizon and both the optimal hedge ratio and hedge effectiveness, using the permanent / transitory representation implied by cointegration. His model implies that the R^2 converges toward one as the hedge horizon increases, while the hedge ratio converges to the ratio of sensitivities of the cash and futures prices, respectively, to the permanent component that drives these prices.

We provide an alternative approach to model the relation between the hedge horizon and the optimal hedge ratio and R^2 , when the underlying price series are cointegrated. We contribute to the dialogue by analyzing a theoretical, prototypical ECM to ascertain the effects of increasing the hedge horizon on the behavior of the model, when the prices are cointegrated. We show that, as the hedge horizon is extended, the error correction coefficient tends toward negative one while the estimated hedge ratio and R^2 both approach positive one. In addition, we prove that the latter two results still hold if the hedger omits the error correction term, and applies the standard textbook approach to estimate a simple regression on price changes.

Our model differs from that of Howard and D'Antonio (1991), by assuming that the cash and futures prices are cointegrated according to a prototypical ECM. Our work offers an alternative approach to the model of Geppert (1995), by assuming that only the cash price adjusts to shocks, while focusing on the behavior of the prototypical ECM itself rather than the permanent / transitory representation implied by the cointegrating system. This approach is valuable since most practitioners who analyze cointegrating relations work in the domain of an ECM rather than the theoretical permanent / transitory representation of the cointegrating system.

Our focus on the nature of the ECM offers a number of insights beyond the analysis of Howard and D'Antonio (1991) and Geppert (1995), by helping us to understand how the specific attributes of the error correction model affect the hedge ratio and R^2 measure. In particular, we show that the coefficient of the error correction term plays a key role in the relation between the hedge horizon and both the hedge ratio and R^2 measure. First, when this coefficient is zero the prices are not cointegrated, the ECM simplifies to the textbook specification, and the optimal hedge ratio and R^2 measure are independent of the hedge horizon. Second, when this coefficient is nonzero but close to zero, the optimal hedge ratio and R^2 converge to one slowly as the hedge horizon is increased. Third, when this coefficient is nonzero, the magnitude of this coefficient itself converges to negative one as the hedge horizon is extended. Fourth, when this coefficient is nonzero but the user does not include the error correction term in the ECM, the estimated hedge ratio and R^2 measure will still increase toward one as the hedge horizon is lengthened.

2. Overview of Hedging, Hedge Accounting, and Econometric Issues

2.1 The Mechanics of Hedging and Hedge Accounting

The use of derivative instruments for hedging is straightforward. For any undesired exposure, find a closely-related derivative instrument (e.g., a futures contract on a similar underlying price) and take a position as an overlay to the exposure being hedged. A well functioning hedge would then be expected to offset the impact of an adverse price move on the exposure. Of course, proper sizing of the hedge position is critical to the success of the effort. However, there is no widespread agreement about the proper methodology for determining the optimal hedge ratio.

This lack of consensus is highlighted by the hedge documentation requirements that the Financial Accounting Standards Board (FASB) has stipulated as prerequisite for applying special hedge accounting treatment. This treatment assures that the earnings recognition from hedging

transactions will be paired in the same accounting period as the earnings impact from the hedged item. Without hedge accounting, these two effects would likely be reported in different periods.

Given the expectation that the derivative will offset the exposure, pairing these two income effects in the same accounting period would tend to lower volatility for reported earnings, as compared to these same earnings impacts being reported in different periods. All else equal, analysts and investors are believed to assign higher stock valuations to companies with lower earnings volatility. Hence the appeal of hedge accounting is understandable.

As appealing as hedge accounting might be, its use is not an election. Rather, hedgers must meet certain requirements to qualify for hedge accounting. In this regard, the finance and accounting literature distinguish between the special case of a perfect hedge versus the general case of a cross-hedge. In a perfect hedge, the *underlying good and location* associated with the exposure are *both* identical to those specified for the hedging derivative. In this case a one-to-one hedge ratio is obvious, no analytics are required, and hedge accounting routinely applies. In contrast, a cross-hedge arises when the exposure and the derivative are not based on the same underlying instrument (price) at the same location. In this case a one-to-one hedge ratio may not be the proper choice. As a result, cross-hedgers are required to conduct a statistical analysis of the hedging relation between the prices of the hedged item and the hedging derivative, to determine the optimal hedge ratio and validate their expectation that the hedge will be “highly effective” in offsetting the fair values or cash flows associated with the risk being hedged.⁸

2.2 *Alternative Regression Models to Estimate Hedge Ratios and Hedge Effectiveness*

In theory, the proper specification for the regression model describing the hedging relation depends on the time series behavior of the prices involved. There are three basic cases:

⁸ This regulation is notoriously complex, yet purposely vague about the specific analysis required. Hedgers must also update the analysis quarterly, throughout the life of the hedge, to show continuing support for their expectation.

1. If the price series do not contain unit roots, then a simple regression on either price levels or price changes may be appropriate.
2. If the price series contain unit roots, but are not cointegrated, then a simple regression on price levels is generally misspecified due to the possibility of spurious regression. In this case a simple regression on price changes (i.e., the textbook approach) is appropriate (Granger and Newbold, 1974).
3. If the price series contain unit roots and are cointegrated, then the textbook approach is misspecified due to omission of relevant variables. In this case the simple regression on price changes can be appended to include an error correction term. The resulting Error Correction Model (ECM) is well-specified in the presence of cointegration.

This study addresses several questions regarding the proper regression design to analyze the hedging relation between the prices of the hedged item and the hedging instrument. In particular, should the regression use price levels or price changes? Is the presence of unit roots and a significant cointegrating relation sufficient to justify the expectation that a hedge will be highly effective? In this case, are the results from estimating a simple regression on price levels or price changes comparable to those from an ECM? For any specification, are the results sensitive to the length of the hedge horizon (i.e., the time frame for measuring price changes)?

In general, improper specification of the regression model may yield unreliable results, casting doubt on the validity of the slope coefficient as the appropriate hedge ratio or the R^2 as an indicator of hedge effectiveness. We are interested in how these three alternative specifications behave when the hedge horizon is lengthened, under conditions of cointegration.

3. Estimating the Hedging Relation Using Price Changes or Price Levels

3.1 Accounting Rules for a Cross-Hedge

Situations involving a cross-hedge are especially common in managing commodity price risk, where cash market transactions are typically priced with a “basis” or differential relative to a readily observable industry reference price, such as the nearby (i.e., next-to-expire) futures price. For example, gasoline pricing follows this practice, where the cash price at any location is

determined relative to the nearby gasoline futures contract traded at the New York Mercantile exchange, which prescribes delivery in New York Harbor. In this case, the underlying prices for the hedged item and the hedging derivative pertain to the same underlying good, but not at the same location. As a result, the basis does not generally converge to zero at maturity.

Prior to the introduction of FAS 133, most hedgers ignored this basis effect in a cross-hedge. That is, gasoline producers elected to hedge what amounted to the “New York Harbor component” of their gasoline price exposure. As long as the associated basis was relatively stable, and/or was only a minor component of the gasoline price (which is typical), these hedgers were largely comfortable that they were addressing the lion’s share of their gasoline price exposure. In these cases the appropriate hedge size remained trivial, and a one-to-one hedge ratio would apply with no regression analysis needed. FAS 133 changed this practice, by restricting hedge accounting treatment only to hedges that strived to offset the full change in price of the gasoline purchases or sales – i.e., not just the New York Harbor portion, but the basis effects as well. Given this restriction, it became necessary for cross-hedgers to determine the hedge size empirically, as the slope coefficient of a regression analysis. Moreover, auditors have been instructed to confirm that the sizing of a hedge position implemented by the hedger is consistent with the regression analysis used by the hedger to assess hedge effectiveness. This latter emphasis on proper hedge sizing is an appropriate consideration, as the R^2 statistic would not be a valid indicator of hedge effectiveness if the size of the hedge implemented was not consistent with the slope of the regression model.

3.2 The Textbook Solution: The Hedging Relation Using Price Changes

The standard textbook approach seeks to minimize uncertainty about changes in the value of the combined hedged position, which includes both the exposure and the derivatives position. This

goal can be accomplished by selecting the hedge ratio (h^*) that minimizes the volatility of changes in the combined hedged portfolio. The solution for h^* is the coefficient generated by a simple regression utilizing price changes, as follows (see Ederington, 1979; Hull, 2008):

$$\Delta C_t = a_C + b_C \Delta F_t + v_t, \quad (1)$$

where ΔC_t = change in the cash price of the hedged item measured over the hedge horizon,
 ΔF_t = change in the futures price of the hedging instrument over the hedge horizon,
 v_t = the regression error term, assumed to be stationary, and independent and identically distributed $N(0, \sigma_v^2)$,
 a_C and b_C are the coefficients from the regression on price changes.

The R^2 from this analysis reflects the proportion of the total variation in cash price changes that is explained by variation in futures price changes. Thus, to the extent that the prospective future relation between changes in cash and futures prices is expected to be similar to that over the historical period analyzed, we can anticipate that using a hedge ratio of b_C should eliminate a proportion of the total risk exposure that is equivalent to this R^2 measure.

3.3 The Hedging Relation Using Price Levels

In the trivial case where the asset representing the risk exposure is identical to the instrument underlying the futures contract, a one-to-one hedge ratio locks in the futures price adjusted for the basis at the time the hedge is liquidated. This result holds irrespective of the price level at which the futures contract happens to be transacted. In this situation, the *level* of the cash price behind the exposure being hedged (C_t) is likely to be highly correlated with the *level* of the futures price on the same underlying asset (F_t).

If this situation applies to the case when the asset representing the exposure and the asset underlying the futures contract are identical, then it is reasonable to assume that it also applies to the more general cross-hedge situation when some basis adjustments may be unrelated to the price changes associated with the industry benchmark prices. As long as the *levels* of the relevant

cash and futures prices are sufficiently highly correlated, the futures contract should be expected to eliminate risk, and thereby serve as an acceptable hedging derivative. This expectation, however, still leaves open the possibility that the basis effects may be systematically related to the underlying benchmark prices, such that a hedge ratio other than one might be appropriate.

In this situation, a demonstration that the two respective price *levels* were sufficiently highly correlated would serve to justify the expectation that the hedge will be highly effective. Thus, it may also be useful and appropriate to formulate the regression analysis as follows:

$$C_t = a_L + b_L F_t + \varepsilon_t, \quad (2)$$

where ε_t is the regression error, assumed to be stationary, and independent and identically distributed $N(0, \sigma_\varepsilon^2)$, while a_L and b_L represent the coefficients for the regression on price levels.

Note that the traditional textbook approach in equation (1) yields a different solution to the problem of determining the proper hedge size and anticipated hedge effectiveness from that in (2). Likewise, the textbook approach is also inconsistent with the ubiquitous practice in the hedging industry of structuring a one-to-one hedge ratio for a cross-hedge, when the asset underlying the futures is identical to the exposure, but the location is not.⁹

With this orientation in mind, many in the accounting world apply the simple regression model specified in either (1) or (2), and set a minimum R^2 value of .80 as a necessary (but not sufficient) condition to satisfy the FAS 133 criterion that a hedge is expected to be highly effective. Once again, for the R^2 statistic from estimating (1) or (2) to have relevance, the hedge ratio implemented by the user would have to be consistent with b_C or b_L , respectively.

⁹ We distinguish between one kind of cross-hedge, where the asset underlying the futures is identical to the exposure but the location is not (e.g., hedging gasoline sales at one location with gasoline futures that prescribe delivery at a different location), versus another kind of cross-hedge where the asset underlying the futures is not identical to the exposure. The industry practice of structuring a one-to-one hedge is prevalent only in the former case.

4. Problems in Applying the Textbook Solution

4.1 Choice of Time Interval for Measuring Price Changes

The textbook solution prescribes analyzing the relation between changes in cash and futures prices, in (1). In conducting this analysis, one critical question regards the appropriate time interval for measuring price changes. The analysis should ideally be limited to the relevant conditions at the start and end of the planned hedge. Thus, the traditional textbook approach recommends applying the regression analysis to data on historical price changes measured over a time interval equal to the hedge horizon (Hull, 2008, pp. 54-58). This choice of time frame for measuring price changes, however, is subject to two significant limitations.

First, it is desirable to have a large number of observations on price changes, ΔC_t and ΔF_t , to achieve the asymptotic properties of regression analysis. In many cases, however, it may not be possible to generate a sample of adequate size, since data on futures prices are frequently unavailable for long horizons. Moreover, it is desirable to limit the analysis to futures prices of the most actively traded contracts, since more liquid contracts are more likely to be efficiently priced. However, futures contracts are often only actively traded toward the end of their lives, while they are the “nearby” (i.e., next-to-expire) contract. For example, although gasoline contracts are currently listed with 18 consecutive monthly expirations, the more distant expiration months are not traded actively. Thus, any effort to construct an extended data set of actively traded futures prices typically involves blending futures prices associated with multiple expirations, by selecting the price of the nearby contract at any point in time.

A second disconcerting implication of the textbook approach has to do with the fact that, as time passes, the hedge horizon for a given hedging problem is ever-diminishing. Current accounting guidance requires hedgers to provide updated analyses at least quarterly throughout the life of the hedge, which are both retrospective and prospective, to document that their hedges

have performed effectively and are expected to continue be effective over their remaining lives.¹⁰ This requirement for ongoing prospective analyses suggests that the hedger should implement a series of regression tests that incorporate price changes measured over time intervals of different lengths, corresponding to a hedge horizon that diminishes over the life of a hedge. For instance, the hedger with a nine-month horizon remaining would use data reflecting nine-month price changes, while three months later the same hedger (now with six months remaining in the hedge horizon) would use six-month price changes in the updated analysis, and so forth.¹¹

These issues lead to the empirical question of whether the nature and extent of the hedging relation (i.e., the slope coefficient and regression R^2) are sensitive to the use of price levels versus changes, and to the time frame used to sample price levels or measure price changes. If the empirical results vary under these respective designs, then it would seem that the hedge ratio should be adjusted throughout the life of a given hedge. However, practitioners generally act as though the nature of the exposure remains unchanged throughout a hedge's life, as they do not adjust hedge ratios during the original hedge horizon.¹²

4.2 Cointegration and the Error Correction Model

4.2.1 The Cointegrating Relation

Consider the following situation in which the cash and futures prices (C_t and F_t) are each integrated of order one, and follow a simple cointegrating relation in which the cash price responds to deviations from equilibrium, while the futures price does not:

¹⁰ This current requirement is also subject to possible changes. The recent exposure draft proposes that the retrospective assessments may only be required if and when there is a material change from the original expectations relating to the hedged item, as of the start of the designated hedging relation.

¹¹ Originally in Derivative Implementation Issue (DIG) E7, reporting entities were granted permission to apply the same regression design for prospective and retrospective testing. This permission has precipitated the common practice of using the same regression specification to serve double duty. This practice, however, reflects an ignorance of the statistical issues relating to the appropriate choice of the time frame for measuring price changes.

¹² There are exceptions to this generalization, such as "tailed hedges" and delta hedging of option positions (see Kawaller, 1997, and Hull, 2008).

$$C_t = \beta F_t + \varepsilon_{1t}, \quad (3)$$

$$\Delta F_t = \varepsilon_{2t}, \quad (4)$$

where ε_{1t} and ε_{2t} are each independent and identically distributed with mean zero and finite variances, σ_1^2 and σ_2^2 , respectively. Moreover, let the covariance between ε_{1t} and ε_{2t} be given by σ_{12} , and let $\beta \neq 0$. Given these assumptions, both C_t and F_t have unit roots. However the difference, $(C_t - \beta F_t) = \varepsilon_{1t}$, is stationary, and C_t and F_t are cointegrated.¹³

In this case the simple regression on price changes in (1) is misspecified by omission of the error correction term. This conclusion challenges the validity of the standard textbook approach, when the price series are cointegrated. Furthermore, we suggest that cash and futures prices are likely to be cointegrated in many cross-hedging situations, due to arbitrage activity.¹⁴

On the other hand, given the cointegrating relation in (3) and (4), a simple regression on price levels, as in (2), also has desirable properties. In particular, note that (2) and (3) are nearly identical specifications of the relation between the price levels. Equation (3) is distinct from (2) in just two ways. First there is no intercept in (3). Intercepts do not have the usual interpretation in cointegrating systems, and omitting the intercept in (3) is an appropriate specification. Second, the presence of (4) serves to link the two price series through σ_{12} , so that they are jointly determined in the cointegrating relation. Thus, the common industry approach of estimating a simple regression on price levels, as in (2), may also provide useful information for the hedger.

4.2.2 *The Error Correction Model (ECM)*

When cash and futures prices are cointegrated as in (3) and (4), it is appropriate to estimate the minimum variance hedge ratio using an ECM. This approach focuses on the relation between

¹³ See Enders (2003, pp. 363-377). Note that the error term in this cointegrating relation, $\varepsilon_{1t} = (C_t - \beta F_t)$, is analogous to the value of the short hedger's combined hedged position, when β is used as the hedge ratio.

¹⁴ For discussion and evidence, see Benz and Hengelbrock (2009), Chou et al. (1996), Ghosh (1993a,b), Kenourgios et al. (2008), Kroner and Sultan (1993), Lien (1996, 2004), Lien and Luo (1993), and Lien and Tse (1999). In contrast, Baillie and Myers (1991) argue that cash and futures prices for commodities may not be cointegrated.

price changes, as in the textbook approach, but it appends (1) to include a lagged error correction term, ε_{1t-1} , along with lagged values of both cash and futures price changes, as follows:

$$\Delta C_t = \alpha_0 + \alpha_1 \varepsilon_{1t-1} + b_{ECM} \Delta F_t + \sum_{k=1}^m \gamma_k \Delta F_{t-k} + \sum_{j=1}^n \delta_j \Delta C_{t-j} + e_t, \quad (5)$$

where b_{ECM} is the optimal hedge ratio, and the error correction term is $\varepsilon_{1t-1} = (C_{t-1} - \beta F_{t-1})$. This term can be determined as the lagged residual from (3), by joint estimation of (3), (4), and (5).¹⁵

In addition, a measure of anticipated hedge effectiveness that pertains to the estimated hedge ratio from the ECM (b_{ECM}) can be constructed as follows:

$$R^2 \text{ Analogue} = 1 - SSE^* / SST^*, \quad (6)$$

where SSE^* = the total variation in the time series, $\{\Delta C_t - b_{ECM} \Delta F_t\}$, about its mean,
and SST^* = the total variation in the time series, $\{\Delta C_t\}$, about its mean.

This measure of hedge effectiveness is analogous to the R^2 from (1), except that SSE^* omits the intercept in (1) and assumes that the hedger uses b_{ECM} as the hedge ratio, rather than b_C .¹⁶

5. The Hedge Horizon, the Hedge Ratio, and the R^2 from a Prototypical ECM

In this section we further examine the simple cointegrating relation in (3) - (5). Specifically, we analyze the effect of increasing the length of the hedge horizon (and thus the time frame for measuring price changes) on the estimated hedge ratio and R^2 from the resulting ECM in (5).

5.1 A Prototypical Error Correction Model

Consider the relation between the cash price of a commodity for delivery to location a at time t (denoted C_{at}) and the futures price (F_t) for future delivery of the same commodity to a different

¹⁵ In this model, enough lags are included (i.e., m and n are selected to be sufficiently large) to ensure that: (i) all relevant information about ΔF_{t-k} and ΔC_{t-i} is incorporated, and (ii) the error term, e_t , is not autocorrelated. In our analysis, the initial choice for m and n is made using the Akaike Criterion. We then apply the Ljung-Box (1978) test on the autocorrelation function of the ECM residuals, to investigate the null hypothesis that e_t is not autocorrelated. If the lag lengths indicated by the Akaike Criterion do not result in white noise residuals, we increase the lag length on the dependent variable (n) further until the Ljung-Box test does not reject the white noise hypothesis.

¹⁶ Note that the R^2 from estimating (1) will be greater than the R^2 analogue in (6), since the R^2 from (1) is the maximum R^2 possible for this situation (although it will likely be less than the R^2 from the ECM in (5) itself).

location. This situation represents a cross-hedge. We posit a joint determination of cash and futures prices at the frequency of observation. In our case study, the commodity is gasoline and the data are observed at the daily frequency. We define the vector of cash and futures prices as:

$$X_t = \begin{pmatrix} C_{at} \\ F_t \end{pmatrix}.$$

Denote $\Delta X_t = X_t - X_{t-1}$, and consider the following assumptions for the joint data-generating process of the cash and futures price series:

Assumption 1: $\Delta X_t = \Pi X_{t-1} + \varepsilon_t$ where ε_t is a martingale difference.

Assumption 2: Suppose that $\Pi = \alpha\beta^T$ with $\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix}$, $\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$,

$$\text{and } E(\varepsilon_t \varepsilon_t^T) = \Omega = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}.$$

Assumption 3: $-1 < \alpha_1 < 0$.

Assumption 4: ε_t is elliptically symmetric.

5.2 How the Assumptions Affect the Behavior of the Prototypical Error Correction Model

The first assumption states that an ECM characterizes the cointegrating relation between the cash and futures price series. The second and third assumptions impose a structure for Π , in which: $-1 < \alpha_1 < 0$, $\alpha_2 = 0$, $\beta_1 = 1$, and $\beta_2 = -1$. The fourth assumption ensures that conditional expectations are linear.¹⁷

These restrictions incorporate several important features for this cointegrating system.

First consider the restrictions on α_1 and α_2 . The restriction on α_1 is sufficient for the difference of the cash price series (ΔC_{at}) to be stationary, and for the system to be cointegrated.¹⁸ Requiring α_2 to be zero implies that the cash price series (C_{at}) responds to any disequilibrium in the relation

¹⁷ This assumption of linear conditional expectations is routine in asset pricing theory. For example, Owen and Rabinovich (1983) show that elliptical symmetry of the error structure is sufficient to generate the CAPM, while Berk (1997) proves that this assumption is not only sufficient, but necessary for the CAPM to be linear.

¹⁸ See Johansen (1996) Ex. 4.1, for the conditions for cointegration and stationarity of the differences in this model.

between the two series, but the futures price series (F_t) does not. If the two prices are cointegrated in this fashion, then they tend to move together in a one-to-one ratio over time. For example, this would likely be the case if arbitrageurs enforce the law of one price in the relation between the cash and futures prices for the same asset at the same location.

Next consider the restrictions on β_1 and β_2 . Together, these restrictions imply that cash price changes (ΔC_{at}) respond to the lagged value of the cash-to-futures basis, $(C_{at-1} - F_{t-1})$. That is, these restrictions imply that the general error correction term, $\varepsilon_{1t-1} = (\beta_1 C_{at-1} + \beta_2 F_{t-1})$, in the ECM simplifies to the cash-to-futures basis, $(C_{at-1} - F_{t-1})$.

For more intuition regarding the behavior of this prototypical ECM, consider a shock that leads to a deviation between cash and futures prices, $(C_{at} - F_t)$, which exceeds the no-arbitrage trading range implied by the cost of carry and transaction costs. Since $-1 < \alpha_1 < 0$ and $\alpha_2 = 0$, the cash price will respond to arbitrage activity induced by such deviations and adjust back toward the equilibrium trading range, but the futures price will not. The restriction on α_2 is often justified in practice, since it implies that F_t does not adjust to past information, but rather depends upon anticipated future information. This cointegrating system characterizes a market in which the futures price serves a price discovery function for cash prices. There is ample support for such price discovery in futures markets.¹⁹

In our case study on gasoline futures and cash prices, we provide empirical support for the restrictions implied by the first three assumptions in this prototypical ECM.

5.3 The Hedge Horizon and the Hedge Ratio, the R^2 , and the Error Correction Term

Given the assumption that the data are observed at a daily frequency, the ECM associated with this cointegrated system applies to daily price changes. However, this daily cointegrated system

¹⁹ For example, see Benz and Hengelbrock (2009), Brandt et al. (2007), Chen and Gau (2009), Kawaller et al. (1987), Quan (1992), Rosenberg and Traub (2006), and Tse et al. (2006).

also implies an analogous ECM when longer time frames are used to measure price changes. We define the N-day price differences as $\Delta_N C_{at}$ ($= C_{at} - C_{at-N}$) and $\Delta_N F_t$ ($= F_t - F_{t-N}$). This notation enables us to examine the implications for estimating this ECM when we increase the time frame for measuring price changes (i.e., when we increase the length of the hedge horizon, N).

5.3.1 The Hedge Horizon and the Optimal Hedge Ratio

For a hedge horizon covering N days ahead, the optimal hedge ratio is calculated by estimating an ECM in which the dependent variable is the N-day change in the cash price ($\Delta_N C_{at}$), and the two key right-hand-side variables are: the N-day lag of the basis ($C_{a,t-N} - F_{t-N}$) and the N-day change in the futures price ($\Delta_N F_t$).²⁰ In general, the optimal hedge ratio for an N-period-ahead hedge is given by the coefficient of $\Delta_N F_t$ in this ECM, which can be characterized as follows:

$$\frac{Cov(\Delta_N C_{at}, \Delta_N F_t | C_{a,t-N} - F_{t-N})}{Var(\Delta_N F_t | C_{a,t-N} - F_{t-N})}.$$

One goal of this paper is to examine the behavior of this optimal hedge ratio as we lengthen the hedge horizon (i.e., as we increase N). To this end, we find an analytical representation of this optimal hedge ratio that is implied by the prototypical ECM, in the following theorem:

Theorem 1: *Suppose that Assumptions 1-3 hold. Then the optimal hedge ratio for N periods ahead is given by:*

$$\left[\frac{1 - (1 + \alpha_1)^N}{N \alpha_1} \right] \left(1 - \frac{\sigma_{12}}{\sigma_2^2} \right) + 1.$$

All proofs are provided in the Appendix.

Given assumption 3 the term, $(1 + \alpha_1)^N$, converges to zero as N increases, so that the optimal hedge ratio will converge to one as we increase the hedge horizon. Moreover, for N = 1, the optimal hedge ratio = σ_{12}/σ_2^2 , which is the classic textbook solution. Note that this

²⁰ In addition, it is standard to include lagged values of futures and cash price changes, $\Delta_N F_{t-k}$ and $\Delta_N C_{t-i}$, as in (5). Note that this specification presumes the use of non-overlapping data on the N-day price changes.

expression does not yield a one-for-one hedge ratio at shorter hedge horizons. To illustrate this point, suppose the following parameter values characterize the cointegrating relation: $\sigma_{12} = 0.5$, $\sigma_2^2 = 1.0$, and $\alpha_1 = -0.1$. Based on these values, the optimal 1-period-ahead hedge ratio implied by the prototypical ECM is 0.5. Then the optimal 2-, 10-, 30-, and 60-period-ahead hedge ratios are 0.525, 0.674, 0.840, and 0.917, respectively.

We also note that, if α_1 is closer to zero, then the optimal hedge ratio converges to one more slowly as we lengthen the hedge horizon. Hence, it is important to estimate the value of α_1 in the ECM, to determine the speed of convergence of the optimal hedge ratio toward one. In particular, many hedgers use a default one-for-one hedge ratio, regardless of the hedge horizon. The value of α_1 determines the potential loss relative to the optimal hedge from such a procedure.

In contrast, if $\alpha_1 = 0$, we have the following corollary:

Corollary 1: *Suppose that Assumptions 1 and 2 hold, but $\alpha_1 = 0$. Then the optimal hedge ratio for N periods ahead is given by:*

$$\frac{\sigma_{12}}{\sigma_2^2}.$$

If $\alpha_1 = 0$, then there is no error correction term in the ECM. With no error correction term, there is no adjustment back toward the equilibrium relation following deviations of the basis outside the no-arbitrage trading range, and thus there is no cointegrating relation. Since the optimal hedge ratio indicated in this corollary does not depend on the length of the hedge horizon (N), we would expect to find that the estimated hedge ratio does not change as we vary the time frame for measuring price changes (i.e., the hedge horizon), when there is no cointegrating relation.

5.3.2 The Hedge Horizon and the Regression R^2

To measure hedge effectiveness for compliance with FAS 133, hedgers may use the R^2 from a regression of $\Delta_N C_{at}$ on $(C_{at-N} - F_{t-N})$ and $\Delta_N F_t$. The following theorem provides an expression for the population R^2 from this regression, which depends on the parameters of our model:

Theorem 2: Suppose that Assumptions 1-4 hold and $\alpha_1 \neq 0$. Then the population R^2 from a regression of $\Delta_N C_{at}$ on $(C_{at-N} - F_{t-N})$ and $\Delta_N F_t$ is given by:

$$R^2 = 1 - \left(\frac{\left(\frac{c_1 + \sigma_2^2}{N} \right) - \frac{\left(\frac{c_2 + \sigma_2^2}{N} \right)^2}{\sigma_2^2}}{\frac{c_3}{N} + \sigma_2^2} \right),$$

where

$$c_1 = \left[\frac{1 - (1 + \alpha_1)^{2N}}{1 - (1 + \alpha_1)^2} \right] (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (2\sigma_2^2 - 2\sigma_{12})$$

$$c_2 = \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12})$$

$$c_3 = c_1 + \left[\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right] [1 - (1 + \alpha_1)^N]^2$$

From this result we see that, as N approaches infinity, the terms, c_1 , c_2 , and c_3 , are bounded so that the R^2 converges to 1. From a practical perspective this result means that, if the ECM is the appropriate model, then hedgers can be confident that their hedge will be effective and their analysis will lead to hedge accounting treatment, if they are willing to consider a sufficiently long hedge horizon.²¹

The following result deals with the case when there is a cointegrating relation, so an error correction term belongs in the ECM, but we fail to include this term in the regression model:

Corollary 2: Suppose that Assumptions 1-4 hold and $\alpha_1 \neq 0$. Then the population R^2 from a simple regression of $\Delta_N C_{at}$ on $\Delta_N F_t$ is given by:

$$R^{*2} = R^2 + \frac{\frac{c_4}{N}}{\frac{c_3}{N} + \sigma_2^2},$$

where R^2 and c_3 are defined as in Theorem 2, and

²¹ Of course, this would require sufficient data on past cash and futures price changes to have ample degrees of freedom to estimate the ECM, as we lengthen the time frame for measuring price changes (i.e., as we increase N).

$$c_4 = \left[(1 + \alpha_1)^N - 1 \right]^2 \left(\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right)$$

This corollary means that, if $\alpha_1 \neq 0$ so that the error correction term belongs in the model, but we exclude it from the regression, the resulting R^2 will still increase toward one as we lengthen the hedge horizon in the same way that it does when we correctly specify the ECM.

On the other hand, if $\alpha_1 = 0$, then we obtain different results for R^2 when we estimate the ECM with and without the error correction term. This result is shown in the following theorem:

Theorem 3: *Suppose that Assumptions 1, 2, and 4 hold, but $\alpha_1 = 0$. Then the population R^2 from a regression of $\Delta_N C_t$ on $(C_{t-N} - F_{t-N})$ and $\Delta_N F_t$ is given by:*

$$\frac{\sigma_{12}^2}{\sigma_1^2 \sigma_2^2}$$

Note that, if $\alpha_1 = 0$, the population R^2 does not depend upon N . Once again, in this situation the price series are not cointegrated and the error correction term does not belong in the regression. Evidently, the adjustment of cash prices to past deviations from equilibrium, via the coefficient α_1 in this cointegrating relation, is the driving force behind Theorems 1 and 2.

5.3.3 The Hedge Horizon and the Coefficient of the Error Correction Term

Corollary 3: *The coefficient of the error correction term in the regression of $\Delta_N C_{at}$ on $(C_{at-N} - F_{t-N})$ and $\Delta_N F_t$ is given by $[(1 + \alpha_1)^N - 1]$.*

Given assumption 3, the term, $(1 + \alpha_1)^N$, converges to zero as N increases, so the coefficient of the error correction term will theoretically converge to negative one. This result implies that the error correction will become more important in the ECM as we lengthen the hedge horizon.

6. A Case Study

We illustrate the implementation of this analysis by examining proprietary data from a firm that uses gasoline futures to hedge the prospective delivery of gasoline at six distinct locations:

Alabama, Chattanooga, Memphis, Nashville East, Nashville West, and Richmond. Basis conditions (i.e., the difference between the cash price at each location and the futures price) are market-determined, such that the basis for each location is established independently. However, sales at each location are hedged with the same futures contract, known as the RBOB contract, which prescribes delivery of gasoline in New York harbor. This situation thus represents a cross-hedge that requires analytics to validate the expectation that the hedges will be effective.

6.1 Data considerations

The starting point and first limitation of the analysis relates to collecting the historical data. We use proprietary data from the company in question, on daily gasoline cash prices for each of the six sales locations listed above. The time frame for the available data is limited to the period extending from January, 2006 through April 20, 2008 (i.e., 578 daily observations). Following prior work, we construct a daily futures price series that is composed of the nearby (i.e., next-to-expire) RBOB contract on any given trading day. These futures prices are rolled over to the next contract after each monthly expiration, on the first business day of the month.

6.2 Tests for Unit Roots and Cointegration

We begin our analysis by conducting preliminary tests for unit roots and cointegration. The unit root tests are presented in Table 1, and the cointegration tests appear in Table 2. We conduct these tests for the price series taken at several different time intervals that correspond to different hedge horizons, including daily, weekly, and monthly. Given our limited sample period, it is not feasible to conduct these asymptotic tests using price series taken at quarterly or 6-month periods, since there are fewer than ten such observations. For each time frame examined beyond one day, our price series constitute the last trading day of the period (i.e., week or month).

The results in Table 1 indicate that the futures price and the six cash price series all contain unit roots, whether measuring price levels at daily, weekly or monthly intervals. That is, regardless of the time frame for measuring price *levels*, we fail to reject the null of a unit root. In contrast, the null is rejected for most cases when price *changes* are measured at daily, weekly, or monthly intervals. This evidence suggests that these price series are integrated of order one.

Results in Table 2 further show that the futures price is significantly cointegrated with the cash prices at all six locations, when daily data are employed. However, the evidence for cointegration is weaker when we use weekly data, as only three of the six weekly cash price series reveal significant cointegrating relations at the .05 level. Furthermore, monthly data reveal no significant cointegrating relation between any cash price and the futures price, at this level of significance. The diminishing statistical significance of these tests for a cointegrating relation at longer time intervals is consistent with a decline in the power of these asymptotic tests, when fewer observations are available.

6.3 The Daily Error Correction Model

In Table 3 we provide the results from estimating the daily prototypical ECM. This daily ECM excludes the contemporaneous change in the futures price that appears in equation (5). This ECM specification is the standard error correction model estimated when the researcher documents a typical cointegrating relation. If the researcher is further interested in estimating the optimal hedge ratio, then the prototypical ECM specification (applied in Table 3) is appended to include the contemporaneous change in the futures price (ΔF_t), as in equation (5). The coefficient of this term (ΔF_t) is the optimal hedge ratio. The appended ECM in (5) then offers an alternative specification to the simple regression in (1), for hedgers wishing to estimate optimal hedge ratios and measure anticipated hedge effectiveness. In our subsequent analysis of the

ECM in Table 6, we follow this procedure by estimating the appended ECM specified in equation (5), to estimate the optimal hedge ratio and R^2 measure, over longer hedge horizons.

In Table 3, we are interested in whether the empirical results for the daily error correction model support the restrictions imposed in our prototypical ECM. Specifically, we test the hypotheses that: (i) $\beta_2 = -1$, (ii) $-1 < \alpha_1 < 0$, and (iii) $\alpha_2 = 0$. Consider each restriction, in turn.

First examine the results for the cointegrating equation, (3), in Panel A of Table 3. The estimate of β_2 is not significantly different from -1, for all six models in Panel A. This outcome indicates that the error correction term, $\varepsilon_{1t-1} = (\beta_1 C_{at-1} + \beta_2 F_{t-1})$, can be replaced with the cash-to-futures basis, $(C_{at-1} - F_{t-1})$, in each ECM estimated. We follow this procedure below in Table 6, when we estimate the analogous ECM in equation (5), over longer hedge horizons.

Second, consider the ECM determining ΔC_{at} , in Panel B of Table 3. These results indicate a rejection of the null hypothesis that $\alpha_1 = 0$, for the error correction models involving all six cash price series. This evidence indicates that the error correction term is an important determinant of cash price changes, in every daily cointegrating relation analyzed. This result implies that these six cash prices respond to past shocks in the equilibrium arbitrage relation between the futures price and each cash price. In addition the estimated coefficient, α_1 , is always between zero and negative one, and is close to zero, consistent with the assumptions of our prototypical ECM. This relatively small estimated value for α_1 implies that the optimal hedge ratio and regression R^2 should both converge slowly toward one, as we increase the number of days in the hedge horizon (i.e., as we increase N).

Third, consider the analogous ECM determining ΔF_t , in Panel C of Table 3. For all six models, we fail to reject the hypothesis that $\alpha_2 = 0$. This evidence contrasts with that in Panel B, indicating that the error correction term is not an important determinant of futures price changes,

in any daily relation analyzed. This result implies that the futures price does not respond to past shocks in the equilibrium relation between the futures price and each cash price. In fact, the individual T-statistics and the overall F-tests in Panel C indicate that there are no significant coefficients at all, in any model for ΔF_t . This evidence supports the specification in (4), in which ΔF_t follows a white noise process. In contrast, the evidence regarding α_1 discussed above suggests that ΔC_{at} does adjust to such shocks. Together, this evidence indicates that the futures price serves a price discovery function in this cointegrating relation.

Fourth, consider the lagged values of ΔF_{t-k} and ΔC_{at-i} in each of the six estimated ECM's for ΔC_{at} and for ΔF_t , respectively, in Panels B and C of Table 3. In every ECM determining ΔC_{at} , there are several lagged coefficients on both ΔF_{t-k} and ΔC_{at-i} that are significant at the .05 level. On the other hand, there are no significant lagged values of ΔF_{t-k} or ΔC_{at-i} in any of the six daily ECM's that determine ΔF_{t-k} . This result provides more evidence that each cash price adjusts slowly to shocks in this cointegrating system, while the futures price is forward-looking.

Finally, at the bottom of Table 3 we provide the determinant of the error structure ($|\Omega|$) and the log likelihood test, which indicate a significant cointegrating relation exists for all six cash price series.

6.4 Analyzing the 3 Regression Specifications to Size Hedges and Measure Hedge Effectiveness

The empirical evidence in Table 3 indicates that our prototypical ECM characterizes the daily cointegrating relation between the gasoline futures price and the cash price of gasoline at all six locations. Next, we shed light on the empirical validity of the analytical implications of our prototypical ECM, by estimating the three regression specifications in equations (1), (2), and (5), respectively, as we vary the hedge horizon and thus the time frame for measuring price changes.

6.4.1 Simple Regression on Price Levels

Table 4 provides the results from estimating the simple regression on price levels in (2). Note that, as we move down successive Panels in Table 4, the time frame for sampling price levels is lengthened and the available sample size diminishes. However, the estimated hedge ratio (b_L) and the measure of hedge effectiveness (R^2) do not vary substantially. Across all six cash prices and all Panels analyzed in Table 3, the estimated slope coefficient, b_L , falls between .78 and .97, while the R^2 ranges between .85 and .999.

For a hedger seeking to infer the “correct” or optimal hedge ratio, one might consider this range of estimated hedge ratios fairly large. We observe, however, that our analysis using price levels over quarterly or six-month periods relies on a very small sample size. Such results should be interpreted with caution, as the asymptotic properties of OLS estimation do not apply.

When we ignore the analyses using quarterly or six-month intervals, and focus instead on the analysis based on at least 25 observations (i.e., using daily, weekly, or monthly observations), the estimates of the hedge ratio lie within a narrow band. For this analysis, the estimated regression model on price levels in (2) reveals consistent, stable implications for this hedging relation, regardless of the point of delivery of the asset or the time frame for taking price levels. In addition, the goodness of fit statistics are uniformly above .80, suggesting an effective hedging relation that would justify hedge accounting treatment for a firm using the gasoline futures contract to hedge cash market exposures at all six locations.

It is important to recall that the regression on price levels is well-specified when the price series have unit roots and are cointegrated, in a fashion consistent with equation (3) from our prototypical ECM. The evidence from Table 3 indicates that this situation applies to the daily

ECM in our case study. Hence, we conclude that the empirical evidence in Table 4 provides relevant information regarding the nature and extent of the hedging relation in equation (3).

6.4.2 Simple Regression on Price Changes

Table 5 presents the analogous results from applying the textbook approach, to estimate a simple regression on price changes as specified in (1). The results differ dramatically from those provided in Table 4, using the simple regression on price levels in (2). In particular, the results are now sensitive to the time frame for measuring price changes.

For example, when daily price changes are analyzed in Panel A of Table 5, we find no evidence of a significant hedging relation for cash price changes at any location, as the estimated hedge ratio (b_C) and R^2 measure are indistinguishable from zero. However, as we proceed down the successive Panels in Table 5 to analyze price changes taken over longer time frames, the magnitude of the estimated hedge ratios and R^2 measures both systematically increase toward one. In fact, when quarterly or six-month intervals are used, the R^2 measures are consistently well above 0.80. This outcome suggests an acceptable level of anticipated hedge effectiveness for these longer hedge horizons. On the other hand, the sample sizes are again too small to offer much confidence, when analyzing price changes measured over quarterly or six-month intervals.

This lack of stability in the estimated hedge ratio and R^2 measure, for analyses based on different time frames, is a concern for proponents of the textbook solution. These results suggest that there is no universally appropriate hedge ratio over the entire life of a gasoline hedge, but rather the hedge size should be adjusted as the remaining hedge horizon varies over time. A strict interpretation of these results would suggest that the hedge ratio should be reduced as time passes and the end of the hedge horizon approaches. In fact, hedging should be terminated well before this end date arrives, since the $R^2 < 0.8$ for hedge horizons less than one quarter.

We do not adhere to such a strict interpretation of these results. Rather, we simply observe that the evidence in Table 5 supports the analytical results established in section 5.3 above. Given that the daily data on gasoline cash and futures prices conform to our prototypical cointegrating relation, Table 5 reveals that the standard textbook approach results in an estimated hedge ratio and R^2 measure that are both substantially less than one when price changes are taken over short time intervals, while both increase toward one as the hedge horizon is lengthened.²²

6.4.3 Error Correction Model

Panels A-D of Table 6 provide the relevant results from estimating the ECM in model (5), as we vary the hedge horizon from daily to weekly, monthly, and quarterly time frames. Insufficient data are available to estimate the error correction model using six-month price changes.

First note that the daily ECM results in Panel A of Table 6 are similar to those in Panel B of Table 3. The minor differences in Panel A of Table 6 are due to two changes in the specification of the ECM estimated in Table 6, relative to that estimated in Panel B of Table 3. In Table 6 we include ΔF_t in the ECM in (5), and we replace the error correction term, ε_{1t-1} , with the cash-to-futures basis, $(C_{at-1} - F_{t-1})$.²³

Second, consider how the coefficient of the error correction term (α_1) varies as we increase the hedge horizon across the Panels in Table 6. As in Panel B of Table 3, Panel A of Table 6 reveals that this coefficient is significantly less than zero but small in magnitude, for daily price changes. As we move down successive Panels in Table 6, to consider longer hedge horizons, this coefficient increases in magnitude toward negative one. In fact, for the monthly and quarterly time intervals in Panels C and D, this coefficient often exceeds negative one in

²² These results are also consistent with prior evidence in other contexts (Ederington, 1979, Geppert, 1995, Howard and D'Antonio, 1991, Hill and Schneeweis, 1982, Kroner and Sultan, 1993, Lee et al., 2009, and Lien, 2000).

²³ This replacement is supported by the result in Panel A of Table 3, indicating a failure to reject $H_0: \beta_2 = -1$.

magnitude, although it is never significantly different from negative one. This evidence supports the analytical result in Corollary 3.

Next we compare the estimated hedge ratios (b_{ECM}) and R^2 measures in Table 6 with the analogous hedge ratios (b_C) and R^2 measures from the simple regression on price changes from Table 5, as we lengthen the hedge horizon. The results from Panels A-C of Table 6 are similar to those provided in Panels A-C of Table 5. This evidence indicates that the optimal hedge ratio and R^2 measure are robust when we estimate a simple regression on price changes as in (1), versus the ECM in (5), to analyze price changes measured at daily, weekly, or monthly intervals.

Panel D of Table 6 reveals somewhat divergent results for the ECM in (5), versus the simple regression in (1) from Panel D of Table 5, when price changes are measured over quarterly intervals. We now find that the estimated hedge ratios (b_{ECM}) from (5) are close to 1.0, which is somewhat larger than the respective hedge ratios (b_C) from (1), which range around 0.70. In addition, now the R^2 analogue is somewhat lower for the quarterly ECM in Table 6. Of course, the ECM estimated in Panel D relies on only 8 quarterly observations.

The empirical evidence in Table 6 corroborates the analytical implications of our prototypical ECM. Given the robust daily cointegrating relation we find between gasoline cash and futures prices, we also find several empirical regularities when we extend the hedge horizon. First, the coefficient of the error correction term grows in magnitude for longer hedge horizons, until it is not significantly different from negative one. Second, the estimated hedge ratio and R^2 measures both increase toward one as we lengthen the time interval for measuring price changes. This evidence indicates that, given a cointegrating relation, the hedger can be confident that the hedge will be effective and the statistical analysis will yield an R^2 that exceeds the threshold of 0.80, if the hedger is willing to consider a sufficiently long hedge horizon. Third, for all hedge

horizons analyzed, the hedge ratio and regression R^2 are virtually identical when we estimate an ECM, and when we omit the error correction term and estimate a simple regression on price changes. This outcome suggests that, if the price series are cointegrated, then it does not matter whether the hedger estimates the ECM in (5), or applies the traditional textbook approach in (1).

Once again, we emphasize that the empirical evidence in this study relies on fewer observations when longer intervals are used to measure price changes. Since the available sample for our case study covers just 27 months, we have non-overlapping data on price changes for just 26 monthly, 8 quarterly, or 3 six-month periods. These small sample sizes make us cautious about generalizing our empirical results, especially for quarterly and six-month horizons. Future research should consider the minimum degrees of freedom that ought to be required in such analysis, before hedgers and regulators rely upon such results. This paucity of data at long horizons represents the major limitation for a hedger conducting this analysis.

7. Summary and Conclusions

This study investigates the relation between the hedge horizon (i.e., the time frame for measuring price changes) and the estimated hedge ratio and regression R^2 , when the price series involved are cointegrated. This analysis contributes to the dialogue on the proper statistical methodology to size hedge positions, and to validate the expectation that a hedging derivative will be effective in offsetting the risk being hedged, in order to aid risk management decisions and qualify for hedge accounting treatment. We compare three alternative regression specifications for estimating hedge ratios and measuring hedge effectiveness: a simple regression on price levels, a simple regression on price changes, and an error correction model (ECM) on price changes.

We analyze how these alternative specifications behave as the hedge horizon is extended, when the prices of the hedged item and the hedging instrument are cointegrated according to a

prototypical ECM. We show that the simple regression on price levels is analogous to one component of the cointegrating system, with results that do not depend upon the time frame for measuring price levels. In contrast, when we analyze a simple regression or ECM on price changes, we prove that the hedge horizon affects the results in a predictable fashion. First, as the hedge horizon is lengthened, the coefficient of the error correction term approaches negative one, while the hedge ratio and R^2 from the ECM increase toward positive one. Second, hedgers will obtain similar results for the hedge ratio and R^2 whether they estimate an ECM, or whether they omit the error correction term and estimate a simple regression on price changes.

These results imply that, if the prototypical ECM characterizes the hedging relation, then it is reasonable to anticipate that the optimal hedge will ultimately be effective in offsetting the exposure. In addition, if data are available for a sufficiently long sample period, then hedgers can be confident in meeting the generally accepted criterion for hedge accounting, if they are willing to consider a long enough hedge horizon. Furthermore, when the price series are cointegrated, it doesn't matter whether hedgers estimate an ECM, or apply the standard textbook approach and estimate a simple regression on price changes.

We provide empirical evidence supporting our analytical results, through a case study that estimates the hedging relation between the gasoline futures price and the cash price of gasoline for delivery at six locations. Initial tests show that the daily gasoline futures price and each daily cash price series behave according to our theoretical, prototypical cointegrating relation. The simple regression on price levels is analogous to one component of the cointegrating system, and it reveals a slope coefficient and R^2 that are both close to one, regardless of the time frame for measuring price levels. In contrast, when we estimate the ECM, we find that a longer time frame for measuring price changes leads to an error correction coefficient that tends toward negative

one, along with a hedge ratio and R^2 that both increase toward positive one. Furthermore, the latter two results also hold for a simple regression on price changes, as well as the ECM.

This analytical and empirical evidence has substantive implications for risk managers and policy-makers. For example, these results suggest that regulators ought to grant special hedge accounting treatment for long term hedgers who can document a significant, stable cointegrating relation that conforms to our prototypical ECM. On the other hand, since these results depend upon the hedge horizon, they raise a number of issues that call for additional research to provide further guidance for risk managers and regulators. For example, more work is needed to assess the asymptotic properties of these estimation techniques, as the hedge horizon (and sample size) is varied. Such analysis could help to ascertain the minimum sample size and level of statistical significance that ought to be required for these tests to justify hedge accounting treatment.

We emphasize that our conclusions strictly pertain only to the prototypical cointegrating relation specified here. This theoretical model imposes stringent conditions on the cash and futures price series being analyzed. We find that the empirical relation between gasoline cash and futures prices analyzed in this study conforms to these stringent conditions. However, these conditions may be too restrictive to characterize many cross-hedging situations that embody both location differences and differences between the assets underlying the hedged item and the hedging instrument. Further research should examine the impact of the hedge horizon on the hedge ratio and R^2 measure, when the cash and futures price series have a cointegrating relation that does not conform to these stringent conditions, or when the price series are not cointegrated.

Finally, this analysis draws attention to unintended consequences that may arise when regulators implement a complex rule that relies upon a specific statistical result, such as a minimum threshold for a regression R^2 , in order to support a desired accounting rule outcome.

Given the complexities that characterize all possible hedging relations which must be analyzed in practice, we suggest that the regulators consider the evidence in this study as they revisit the accounting guidance for risk management activities that pertain to special hedge accounting treatment. We hope that consideration of these issues leads to further advances toward coherent and efficacious rules that promote healthy risk management and accounting practices.

Appendix: Proofs

Three lemmas are required to prove our analytical results. We first present each lemma, in turn.

Lemma 1: Suppose that Assumption 1 holds. Then :

$$\Delta_N X_t = \left[\sum_{i=0}^{N-1} \Pi(\Pi + I)^i \right] X_{t-N} + \sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i}$$

Proof: We prove this lemma by induction. For $N=2$, we have:

$$\begin{aligned} \Delta_2 X_t &= \Delta X_t + \Delta X_{t-1} \\ &= \Pi X_{t-1} + \varepsilon_t + \Pi X_{t-2} + \varepsilon_{t-1} \\ &= \Pi[(\Pi + I)X_{t-2} + \varepsilon_{t-1}] + \varepsilon_t + \Pi X_{t-2} + \varepsilon_{t-1} \\ &= \left[\sum_{i=0}^1 \Pi(\Pi + I)^i \right] X_{t-2} + \sum_{i=0}^1 (\Pi + I)^i \varepsilon_{t-i} \end{aligned}$$

so that the statement holds for $N=2$. Now suppose that the statement holds for $N=M$. We show that this implies that it holds for $M+1$. Then:

$$\begin{aligned} \Delta_{M+1} X_t &= \Delta_M X_t + \Delta X_{t-M} \\ &= \left[\sum_{i=0}^{M-1} \Pi(\Pi + I)^i \right] X_{t-M} + \sum_{i=0}^{M-1} (\Pi + I)^i \varepsilon_{t-i} + \Pi X_{t-M-1} + \varepsilon_{t-M} \\ &= \left[\sum_{i=0}^{M-1} \Pi(\Pi + I)^i \right] [(\Pi + I)X_{t-M-1} + \varepsilon_{t-M}] + \sum_{i=0}^{M-1} (\Pi + I)^i \varepsilon_{t-i} + \Pi X_{t-M-1} + \varepsilon_{t-M} \\ &= \left[\sum_{i=0}^{M-1} \Pi(\Pi + I)^{i+1} + \Pi \right] X_{t-M-1} + \sum_{i=0}^{M-1} (\Pi + I)^i \varepsilon_{t-i} + \left[\sum_{i=0}^{M-1} \Pi(\Pi + I)^i + I \right] \varepsilon_{t-M} \\ &= \left[\sum_{i=0}^M \Pi(\Pi + I)^i \right] X_{t-M-1} + \sum_{i=0}^M (\Pi + I)^i \varepsilon_{t-i} \end{aligned}$$

where the last equality comes from:

$$\left[\sum_{i=0}^{M-1} \Pi(\Pi + I)^i \right] = -[I - (\Pi + I)^M],$$

so that the induction statement is true.

Lemma 2:
$$\begin{pmatrix} 1 + \alpha_1 & -\alpha_1 \\ 0 & 1 \end{pmatrix}^N = \begin{pmatrix} (1 + \alpha_1)^N & 1 - (1 + \alpha_1)^N \\ 0 & 1 \end{pmatrix}$$

Proof: The statement is obvious for $N=2$. Suppose that it holds for $N=M$. Then we have:

$$\begin{aligned}
\begin{pmatrix} 1+\alpha_1 & -\alpha_1 \\ 0 & 1 \end{pmatrix}^{M+1} &= \begin{pmatrix} (1+\alpha_1)^M & 1-(1+\alpha_1)^M \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1+\alpha_1 & -\alpha_1 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} (1+\alpha_1)^{M+1} & -\alpha_1(1+\alpha_1)^M + 1-(1+\alpha_1)^M \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} (1+\alpha_1)^{M+1} & 1-(1+\alpha_1)^{M+1} \\ 0 & 1 \end{pmatrix}
\end{aligned}$$

so that Lemma 2 is true by induction.

Lemma 3: Define $v_{(N)t} = \sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i}$ and $\Omega_{(N)} = E(v_{(N)} v_{(N)}^T) = \begin{pmatrix} \sigma_{(N)1}^2 & \sigma_{(N)12} \\ \sigma_{(N)12} & \sigma_{(N)2}^2 \end{pmatrix}$,

and suppose that Assumptions 1-3 hold. Then:

$$\begin{aligned}
\sigma_{(N)1}^2 &= \left[\frac{1-(1+\alpha_1)^{2N}}{1-(1+\alpha_1)^2} \right] (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + \left[\frac{1-(1+\alpha_1)^N}{\alpha_1} \right] (2\sigma_2^2 - 2\sigma_{12}) + \sigma_2^2 N \\
\sigma_{(N)12} &= \left[\frac{1-(1+\alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12}) + \sigma_2^2 N \\
\sigma_{(N)2}^2 &= \sigma_2^2 N
\end{aligned}$$

Proof:

$$\begin{aligned}
\Omega_{(N)} &= E \left[\sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i} \right] \left[\sum_{j=0}^{N-1} \varepsilon_{t-j}^T (\Pi^T + I)^j \right] \\
&= \sum_{i=0}^{N-1} (\Pi + I)^i \Omega (\Pi^T + I)^i \\
&= \sum_{i=0}^{N-1} \begin{pmatrix} (1+\alpha_1)^i & 1-(1+\alpha_1)^i \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \begin{pmatrix} (1+\alpha_1)^i & 0 \\ 1-(1+\alpha_1)^i & 1 \end{pmatrix} \\
&= \sum_{i=0}^{N-1} \begin{pmatrix} (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})(1+\alpha_1)^{2i} + 2(\sigma_{12} - \sigma_2^2)(1+\alpha_1)^i + \sigma_2^2 & \dots \\ (\sigma_{12} - \sigma_2^2)(1+\alpha_1)^i + \sigma_2^2 & \sigma_2^2 \end{pmatrix} \\
&= \begin{pmatrix} \left[\frac{1-(1+\alpha_1)^{2N}}{1-(1+\alpha_1)^2} \right] (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + \left[\frac{1-(1+\alpha_1)^N}{\alpha_1} \right] (2\sigma_2^2 - 2\sigma_{12}) + \sigma_2^2 N & \dots \\ \left[\frac{1-(1+\alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12}) + \sigma_2^2 N & \sigma_2^2 N \end{pmatrix}.
\end{aligned}$$

We are now ready to prove the theorems.

Theorem 1: Suppose that Assumptions 1-3 hold. Then the optimal hedge ratio for N periods ahead is given by :

$$\left[\frac{1 - (1 + \alpha_1)^N}{N \alpha_1} \right] \left(1 - \frac{\sigma_{12}}{\sigma_2^2} \right) + 1.$$

Proof of Theorem 1: Let $X_t = \begin{pmatrix} C_t \\ F_t \end{pmatrix}$. Then by Lemma 1 we have:

$$\Delta_N X_t = \left[\sum_{i=0}^{N-1} \Pi(\Pi + I)^i \right] X_{t-N} + \sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i}.$$

Define $v_{(N)t} = \sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i}$, and let $\Omega_{(N)} = E(v_{(N)} v_{(N)}^T) = \begin{pmatrix} \sigma_{(N)1}^2 & \sigma_{(N)12} \\ \sigma_{(N)12} & \sigma_{(N)2}^2 \end{pmatrix}$.

Then the optimal hedge ratio is given by:

$$\begin{aligned} \frac{\text{Cov}(\Delta_N C_t, \Delta_N F_t | C_{t-N}, F_{t-N})}{\text{Var}(\Delta_N F_t | C_{t-N}, F_{t-N})} &= \frac{\sigma_{(N)12}}{\sigma_{(N)2}^2} \\ &= \frac{\left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12}) + \sigma_2^2 N}{\sigma_2^2 N} \\ &= \left[\frac{1 - (1 + \alpha_1)^N}{N \alpha_1} \right] \left(1 - \frac{\sigma_{12}}{\sigma_2^2} \right) + 1 \end{aligned}$$

Theorem 2: Suppose that Assumptions 1-4 hold. Then the population R^2 from regressing $\Delta_N C_{at}$ on $(C_{at-N} - F_{t-N})$ and $\Delta_N F_t$ is given by:

$$1 - \left(\frac{\left(\frac{c_1 + \sigma_2^2}{N} \right) - \frac{\left(\frac{c_2 + \sigma_2^2}{N} \right)^2}{\sigma_2^2}}{\frac{c_3 + \sigma_2^2}{N} + \sigma_2^2} \right),$$

where

$$\begin{aligned} c_1 &= \left[\frac{1 - (1 + \alpha_1)^{2N}}{1 - (1 + \alpha_1)^2} \right] (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (2\sigma_2^2 - 2\sigma_{12}) \\ c_2 &= \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12}) \\ c_3 &= c_1 + \left[\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right] \left[1 - (1 + \alpha_1)^N \right]^2 \end{aligned}$$

Proof of Theorem 2: Let $w_{(N)t}$ be the residual from regressing $\Delta_N C_t$ on $(C_{t-N} - F_{t-N})$ and $\Delta_N F_t$. Then the population R^2 is given as:

$$1 - \frac{E(w_{(N)t}^2)}{E(\Delta_N C_t)^2}.$$

Since ε_t is elliptically symmetric, the conditional expectation of $v_{(N)1t}$ given $\Delta_N X_{2t} = v_{(N)2t}$ is

linear. Hence, we have $w_{(N)t} = v_{(N)1t} - \frac{\sigma_{(N)12}}{\sigma_{(N)2}^2} v_{(N)2t}$, so that:

$$\begin{aligned} E(w_{(N)t}^2) &= E\left[\left(v_{(N)1t} - \frac{\sigma_{(N)12}}{\sigma_{(N)2}^2} v_{(N)2t}\right)^2\right] \\ &= \sigma_{(N)1}^2 + \frac{\sigma_{(N)12}^2}{\sigma_{(N)2}^4} \sigma_{(N)2}^2 - 2 \frac{\sigma_{(N)12}}{\sigma_{(N)2}^2} \sigma_{(N)12} \\ &= \sigma_{(N)1}^2 - \frac{\sigma_{(N)12}^2}{\sigma_{(N)2}^2} \end{aligned}$$

The model is given as:

$$\begin{aligned} \Delta_N X_t &= \left[\sum_{i=0}^{N-1} \Pi(\Pi + I)^i \right] X_{t-N} + \sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i} \\ &= \left[\sum_{i=0}^{N-1} \Pi(\Pi + I)^i \right] X_{t-N} + v_{(N)t} \end{aligned}$$

From Lemma 2, we have:

$$\begin{aligned} \sum_{i=0}^{N-1} \Pi(\Pi + I)^i &= \sum_{i=0}^{N-1} \begin{pmatrix} \alpha_1 & -\alpha_1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} (1 + \alpha_1)^i & 1 - (1 + \alpha_1)^i \\ 0 & 1 \end{pmatrix} \\ &= \sum_{i=0}^{N-1} \begin{pmatrix} \alpha_1(1 + \alpha_1)^i & -\alpha_1(1 + \alpha_1)^i \\ 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} (1 + \alpha_1)^N - 1 & 1 - (1 + \alpha_1)^N \\ 0 & 0 \end{pmatrix} \end{aligned}$$

so that the first row of this model can be written as:

$$\Delta_{(N)} C_t = \left[(1 + \alpha_1)^N - 1 \right] (C_{t-N} - F_{t-N}) + v_{(N)1t}.$$

Then $E(\Delta_N C_t)^2 = \left[(1 + \alpha_1)^N - 1 \right]^2 \text{Var}(C_{t-N} - F_{t-N}) + \sigma_{(N)1}^2$, since $v_{(N)t}$ is uncorrelated with $C_{t-N} - F_{t-N}$. The proof is completed by finding the final variance. Note from the model that we can write $X_t = (\Pi + I)X_{t-1} + \varepsilon_t$ so that:

$$\begin{aligned}
\beta^T X_t &= \beta^T (\Pi + I) X_{t-1} + \beta^T \varepsilon_t \\
&= (1 \quad -1) \begin{pmatrix} 1 + \alpha_1 & -\alpha_1 \\ 0 & 1 \end{pmatrix} X_{t-1} + \beta^T \varepsilon_t \\
&= (1 + \alpha_1) \beta^T X_{t-1} + \beta^T \varepsilon_t
\end{aligned}$$

where the last equality comes from Assumption 2. By Assumption 3, this is a stationary autoregressive model so that the variance of $\beta^T X_t$ is given by:

$$\frac{\text{Var}(\beta^T \varepsilon_t)}{1 - (1 + \alpha_1)} = \frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1}$$

Combining terms, the population R^2 is given by:

$$1 - \left(\frac{\sigma_{(N)1}^2 - \frac{\sigma_{(N)12}^2}{\sigma_{(N)2}^2}}{\left[\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right] \left[1 - (1 + \alpha_1)^N \right]^2 + \sigma_{(N)1}^2} \right)$$

Substituting the values for $\sigma_{(N)1}^2, \sigma_{(N)2}^2, \sigma_{(N)12}$ gives:

$$1 - \left(\frac{\left(\frac{c_1 + \sigma_2^2}{N} \right) - \frac{\left(\frac{c_2}{N} + \sigma_2^2 \right)^2}{\sigma_2^2}}{\frac{c_3 + \sigma_2^2}{N}} \right),$$

with:

$$c_1 = \left[\frac{1 - (1 + \alpha_1)^{2N}}{1 - (1 + \alpha_1)^2} \right] (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (2\sigma_2^2 - 2\sigma_{12})$$

$$c_2 = \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12})$$

$$c_3 = c_1 + \left[\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right] \left[1 - (1 + \alpha_1)^N \right]^2$$

Corollary 2: Suppose that Assumptions 1-4 hold and $\alpha_1 \neq 0$. Then the population R^2 from regressing $\Delta_N C_{at}$ on $\Delta_N F_t$ is given by:

$$R^{*2} = R^2 + \frac{\frac{c_4}{N}}{\frac{c_3}{N} + \sigma_2^2},$$

where

$$c_4 = \left[(1 + \alpha_1)^N - 1 \right]^2 \left(\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right),$$

and R^2 is defined as in Theorem 2. This corollary means that, if the error correction term belongs in the model but is omitted, the regression R^2 will still approach one as N increases.

Proof of Corollary 2: Let $w_{(N)t}^*$ be the residual from regressing $\Delta_N C_t$ on only $\Delta_N F_t$. Then the population R^{*2} is given as:

$$1 - \frac{E(w_{(N)t}^{*2})}{E(\Delta_N C_t)^2}.$$

Since ε_t is elliptically symmetric, the conditional expectation of $v_{(N)1t}$ given $\Delta_N X_{2t} = v_{(N)2t}$ is linear. From the relationship $\Delta_{(N)} C_t = \left[(1 + \alpha_1)^N - 1 \right] (C_{t-N} - F_{t-N}) + v_{(N)1t}$ and the fact that the expectation of $(C_{t-N} - F_{t-N})$ does not depend on $(F_t - F_{t-N})$, we have:

$$w_{(N)t}^* = \left[(1 + \alpha_1)^N - 1 \right] (C_{t-N} - F_{t-N}) + v_{(N)1t} - \frac{\sigma_{(N)12}}{\sigma_{(N)2}^2} v_{(N)2t},$$

and the result follows from the same arguments in Theorem 2.

Theorem 3: Suppose that Assumptions 1, 2, and 4 hold, but $\alpha_1 = 0$. Then the population R^2 from regressing $\Delta_N C_t$ on $(C_{t-N} - F_{t-N})$ and $\Delta_N F_t$ does not depend upon N , and is always given by:

$$\frac{\sigma_{12}^2}{\sigma_1^2 \sigma_2^2}.$$

Proof of Theorem 3: If $\alpha_1 = 0$, then:

$$\begin{aligned} \Omega_{(N)} &= E \left[\sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i} \right] \left[\sum_{j=0}^{N-1} \varepsilon_{t-j}^T (\Pi^T + I)^j \right] \\ &= \sum_{i=0}^{N-1} (\Pi + I)^i \Omega (\Pi^T + I)^i \\ &= \sum_{i=0}^{N-1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} N\sigma_1^2 & N\sigma_{12} \\ N\sigma_{12} & N\sigma_2^2 \end{pmatrix} \end{aligned}$$

and $\Delta_{(N)} C_t = v_{(N)1t}$ so that the population R^2 is:

$$1 - \left(\frac{\left[\sigma_{(N)1}^2 - \frac{\sigma_{(N)12}^2}{\sigma_{(N)2}^2} \right]}{\sigma_{(N)1}^2} \right) = \frac{\sigma_{12}^2}{\sigma_1^2 \sigma_2^2}.$$

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Table 1. Unit Root Tests on Futures and Cash Prices for Gasoline

In the Panels below, we present unit root tests for gasoline futures and cash prices over the period, Jan. 1, 2006 through Apr. 20, 2008. The unit root tests are conducted on each spot price, as well as on the futures price. The null hypothesis is that there *is* a unit root. The test is based on the Elliott, Rothenberg, and Stock (1996) version of the Augmented Dickey Fuller test. For the spot prices, the test is based on the model:

$$\Delta C_{at} = \gamma_0 + \gamma_1 t + (\rho - 1)C_{at-1} + \beta_1 \Delta C_{at-1} + \beta_2 \Delta C_{at-2} + \dots + \beta_k \Delta C_{at-k} + \varepsilon_t,$$

where k is the number of lagged differenced terms which is determined by the Modified Akaike Criterion of Ng and Perron (2001). We give the unit root tests for the futures price and for the cash prices at six locations. The locations are:

- $a = 1$ (Alabama), 2 (Chattanooga), 3 (Memphis),
 4 (Nashville East), 5 (Nashville West), and 6 (Richmond).

The 1%, 5%, and 10% critical values for the test are -3.97, -3.41, and -3.12, respectively. A t -statistic smaller than this number implies a failure to reject the null hypothesis of a unit root. Note that we only provide results of the unit root tests for the data measured at daily, weekly, and monthly frequencies. The analogous tests are not conducted for longer time intervals, since there are only nine quarterly observations and only four 6-month intervals during our sample period.

Panel A. Daily Price Level (N = 578 days)

Price:	F	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
T-stat	-1.56	-1.81	-1.62	-1.52	-1.73	-1.71	-1.61

Panel B. Daily Price Change (N = 577 daily changes)

Δ Price:	ΔF	ΔC_1	ΔC_2	ΔC_3	ΔC_4	ΔC_5	ΔC_6
T-stat	-5.17***	-4.03***	-3.99***	-4.45***	-4.52***	-4.44***	-4.27***

Panel C. Weekly Price Level (N = 120 weeks)

Price:	F	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
T-stat	-1.35	-1.74	-1.91	-1.84	-1.83	-1.92	-1.65

Panel D. Weekly Price Change (N = 119 weekly changes)

Δ Price:	ΔF	ΔC_1	ΔC_2	ΔC_3	ΔC_4	ΔC_5	ΔC_6
T-stat	-3.28*	-4.13***	-4.18***	-4.29***	-3.02	-3.28*	-2.74

Panel E. Monthly Price Level (N = 27 months)

Price:	F	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
T-stat	-1.10	-1.01	-1.02	-0.99	-1.00	-0.80	-0.91

Panel F. Monthly Price Change (N = 26 monthly changes)

Δ Price:	ΔF	ΔC_1	ΔC_2	ΔC_3	ΔC_4	ΔC_5	ΔC_6
T-stat	-2.93	-3.97***	-3.99***	-3.88**	-3.88**	-3.57**	-3.78**

* indicates statistical significance at the .10 level; ** at the .05 level; and *** at the .01 level.

Table 2. Cointegration Tests on Futures and Cash Prices for Gasoline

In the Panels below, we provide Johansen’s (1996) test for cointegration based on the bivariate Vector Autoregression given by:

$$\Delta X_t = \mu + \gamma t + \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \dots + \Gamma_k \Delta X_{t-k} + \varepsilon_t$$

Where $X_t = (C_{at}, F_t)'$, the vector containing the gasoline cash and futures price at time t. The rank of Π determines the number of cointegrating vectors. The number of lags, k, is determined by the multivariate Akaike Criterion. The null hypothesis is that no cointegrating vectors exist. Rejection of the null implies cointegration. The 5% percent critical value of the Johansen’s Trace Statistic is 25.87. We give the Trace Statistic and its associated p-values for this cointegration test for the relation between gasoline futures and cash prices at six locations. The locations are: a = 1 (Alabama), 2 (Chattanooga), 3 (Memphis), 4 (Nashville East), 5 (Nashville West), and 6 (Richmond).

We only provide results of the cointegration tests for the data measured at daily, weekly, and monthly frequencies. The analogous tests are not conducted for longer time intervals, since there are only nine quarterly observations, and four six-month intervals during our sample period.

Panel A. Daily Price Level (N = 578 days)

Cash Price:	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
Trace Test	38.35***	49.08***	78.25***	34.34***	34.55***	77.30***
p-value	0.0009	0.0000	0.0000	0.0034	0.0032	0.0000

Panel B. Weekly Price Level (N = 120 weeks)

Cash Price:	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
Trace Test	27.45**	23.07	28.09**	19.39	19.72	36.91***
p-value	0.0316	0.1074	0.0261	0.2584	0.2402	0.0014

Panel C. Monthly Price Level (N = 27 months)

Cash Price:	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
Trace Test	19.57	19.30	19.47	19.68	20.53	18.73
p-value	0.2479	0.2635	0.2542	0.2425	0.2005	0.2971

* indicates statistical significance at the .10 level; ** at the .05 level; and *** at the .01 level.

Table 3. Estimated Daily Cointegrating Relation and Error Correction Model

Panel A. Cointegrating Relation for Daily Level of Cash Price, C_{at} , from Equation (3)

Dependent Variable:	C_1	C_2	C_3	C_4	C_5	C_6
Intercept	-.59	-.58	-.53	-.59	-.60	-.48
C_{at-1} (β_1)	1.0	1.0	1.0	1.0	1.0	1.0
F_{t-1} (β_2)	-.982	-.986	-.996	-.992	-.994	-1.025
std error(β_2)	(.039)	(.035)	(.030)	(.047)	(.049)	(.025)
T ($H_0: \beta_2 = -1$)	-.46	-.40	-.13	-.17	-.12	.98

Panel B. Error Correction Model for Daily Change in Cash Price, ΔC_{at} , analogous to (5)

Dependent Variable:	ΔC_1	ΔC_2	ΔC_3	ΔC_4	ΔC_5	ΔC_6
Intercept	.0007	.0008	.0008	.0006	.0006	.0009
ε_{1t-1} (α_1)	-.042	-.050	-.049	-.031	-.028	-.065
std error(α_1)	(.007)	(.008)	(.006)	(.006)	(.005)	(.009)
T ($H_0: \alpha_1 = 0$)	-5.82***	-6.24***	-7.61***	-5.44***	-5.69***	-7.35***
[m, n] ^a [L ₁ , L ₂] ^b	[5,5] [3,2]	[5,5] [2,2]	[3,3] [1,2]	[5,5] [2,4]	[4,4] [2,3]	[4,4] [2,2]
Adjusted R ²	.49	.45	.58	.54	.52	.58
Overall F-test	50.4***	43.8***	116.0***	63.1***	70.1***	87.7***

Panel C. Error Correction Model for Daily Change in Futures Price, ΔF_t , analogous to (5)

Dependent Variable:	ΔF	ΔF	ΔF	ΔF	ΔF	ΔF
Intercept	.0022	.0020	.0021	.0021	.0021	.0021
ε_{1t-1} (α_2)	-.022	-.006	-.023	-.016	-.017	-.014
std error(α_2)	(.027)	(.028)	(.026)	(.024)	(.022)	(.033)
T ($H_0: \alpha_2 = 0$)	-.81	-.21	-.90	-.68	-.78	-.42
[m, n] ^a [L ₁ , L ₂] ^b	[5,5] [0,0]	[5,5] [0,0]	[3,3] [0,0]	[5,5] [0,0]	[4,4] [0,0]	[4,4] [0,0]
Adjusted R ²	.02	-.01	.00	-.01	-.01	-.01
Overall F-test	.89	.46	.79	.58	.66	.56

Panel D. Determinant of the Covariance Structure between ε_{1t} and ε_{2t} , and Log Likelihood Test

$ \Omega $	4.0E-07	4.5E-07	3.4E-07	3.2E-07	2.8E-07	3.8E-07
log likelihood	2605.4***	2570.6***	2657.6***	2669.2***	2705.4***	2619.0***

* indicates statistical significance for each T-statistic at the .10 level; ** at the .05 level; and *** at the .01 level.

^a This row provides lag lengths for the ECM (m and n), indicated by the Akaike Criterion and the Ljung-Box test.

^b This row presents the number of *significant* lagged coefficients on ΔC_{at-i} (L₁), and the number of *significant* lagged coefficients on ΔF_{t-k} (L₂), in the ECM at the .05 level.

Table 4. Measuring the Nature and Extent of the Hedging Relation between the Levels of Futures and Cash Prices over Different Time Frames: Simple Regression Model

In this Table, we analyze data on the levels of spot and futures prices for gasoline over the period, Jan. 1, 2006 through Apr. 20, 2008. In each Panel we analyze price levels taken at different time intervals including daily, weekly, monthly, quarterly, and six-month periods. We summarize the relevant results from the following simple regression model:

Hedging Relation, Price Levels: $C_{at} = a_L + b_L F_t + v_t$, (2)

where C_{at} = daily closing spot price of gasoline at location a at time t;

F_t = daily closing futures price of gasoline (RBOB) at time t;

with location a = 1 (Alabama), 2 (Chattanooga), 3 (Memphis), 4 (Nashville East), 5 (Nashville West), and 6 (Richmond).

Panel A. Daily Price Level (N = 578 days)

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
b _L	.88	.89	.89	.87	.86	.94
T-stat	(66.6)***	(70.2)***	(69.2)***	(60.6)***	(53.7)***	(79.6)***
R ²	.89	.90	.89	.87	.85	.92

Panel B. Weekly Price Level (closing prices on last day each week; N = 120 weeks)

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
b _L	.89	.90	.90	.87	.86	.95
T-stat	(31.5)***	(32.9)***	(32.6)***	(28.5)***	(26.7)***	(38.0)***
R ²	.89	.90	.90	.88	.86	.93

Panel C. Monthly Price Level (closing prices on last day each month; N = 27 months)

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
b _L	.91	.91	.93	.89	.87	.97
T-stat	(13.3)***	(14.5)***	(14.3)***	(12.0)***	(11.6)***	(16.5)***
R ²	.88	.90	.90	.86	.85	.92

Panel D. Quarterly Price Level (closing prices on last day each quarter; N = 9 qtrs)

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
b _L	.84	.85	.86	.80	.78	.95
T-stat	(8.7)***	(8.8)***	(10.0)***	(7.8)***	(6.8)***	(11.6)***
R ²	.93	.93	.94	.91	.91	.96

Panel E. Six-Month Price Level (closing prices on last day each 6-month pd; N = 4)

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
b _L	.79	.78	.83	.82	.82	.82
T-stat	(43.6)***	(24.6)***	(33.2)***	(12.1)***	(7.6)***	(12.8)***
R ²	.9995	.9983	.9991	.9932	.9830	.9939

* indicates statistical significance at the .10 level; ** at the .05 level; and *** at the .01 level.

Table 5. Measuring the Nature and Extent of the Hedging Relation between Changes in Futures and Cash Prices over Different Time Frames: Simple Regression Model

In the different Panels below, we analyze data on changes in spot and futures prices for gasoline over the period, Jan. 1, 2006 through Apr. 20, 2008, using non-overlapping data on price changes taken over daily, weekly, monthly, quarterly, and six-month periods. We summarize the relevant results from the following simple regression model:

Hedging Relation, Price Changes:
$$\Delta_N C_{at} = a_C + b_C \Delta_N F_t + \varepsilon_t, \quad (1)$$

where $\Delta_N C_{at} = C_{at} - C_{at-N}$ = change in cash price over N-day interval;

$\Delta_N F_t = F_t - F_{t-N}$ = change in futures price over N-day interval;

with N = approximate number of days in period of difference; weekly ($N = 5$), monthly ($N = 21$), quarterly ($N = 63$), 6-months ($N = 126$);

and location a = 1 (Alabama), 2 (Chattanooga), 3 (Memphis), 4 (Nashville East), 5 (Nashville West), and 6 (Richmond).

Panel A. Daily Change: (N = 577 daily changes)

	$\Delta_1 C_1$	$\Delta_1 C_2$	$\Delta_1 C_3$	$\Delta_1 C_4$	$\Delta_1 C_5$	$\Delta_1 C_6$
b_C	.03	.02	.01	.01	.01	-.004
T-stat	(1.7)	(1.3)	(0.5)	(0.5)	(1.0)	(-0.2)
R^2	.005	.003	.000	.001	.002	.000

Panel B. Weekly Change: (between closing prices on last day each week; N = 119)

	$\Delta_5 C_1$	$\Delta_5 C_2$	$\Delta_5 C_3$	$\Delta_5 C_4$	$\Delta_5 C_5$	$\Delta_5 C_6$
b_C	.26	.26	.27	.24	.23	.30
T-stat	(4.4)***	(4.3)***	(4.2)***	(4.1)***	(4.1)***	(4.6)***
R^2	.14	.13	.13	.13	.13	.15

Panel C. Monthly Change: (between closing prices on last day each month; N = 26)

	$\Delta_{21} C_1$	$\Delta_{21} C_2$	$\Delta_{21} C_3$	$\Delta_{21} C_4$	$\Delta_{21} C_5$	$\Delta_{21} C_6$
b_C	.62	.64	.66	.58	.55	.71
T-stat	(4.7)***	(5.1)***	(5.1)***	(4.2)***	(4.3)***	(5.3)***
R^2	.48	.52	.52	.43	.44	.54

Panel D. Quarterly Change: (between closing prices on last day each week; N = 8)

	$\Delta_{63} C_1$	$\Delta_{63} C_2$	$\Delta_{63} C_3$	$\Delta_{63} C_4$	$\Delta_{63} C_5$	$\Delta_{63} C_6$
b_C	.67	.69	.70	.61	.58	.83
T-stat	(7.3)***	(6.9)***	(8.7)***	(6.2)***	(4.9)***	(8.1)***
R^2	.90	.89	.93	.86	.80	.92

Panel E. Six-Month Change: (between closing prices on last day every 6-months; N = 3)

	$\Delta_{121} C_1$	$\Delta_{121} C_2$	$\Delta_{121} C_3$	$\Delta_{121} C_4$	$\Delta_{121} C_5$	$\Delta_{121} C_6$
b_C	.70	.70	.75	.74	.74	.79
T-stat	(14.2)***	(11.4)***	(514.6)***	(7.4)***	(5.0)***	(13.9)***
R^2	.995	.992	1.00	.982	.961	.995

* indicates statistical significance at the .10 level; ** at the .05 level; and *** at the .01 level.

Table 6. Measuring the Nature and Extent of the Hedging Relation between Changes in Futures and Cash Prices over Different Time Frames: Error Correction Model

In the Panels below, we analyze data on changes in spot and futures prices for gasoline over the period, Jan. 1, 2006 through Apr. 20, 2008, using the following Error Correction Model:

$$\Delta_k C_{at} = \alpha_0 + \alpha_1 \varepsilon_{1t-N} + b_{ECM} \Delta_N F_t + \sum_{i=1}^m \gamma_i \Delta_N F_{t-i} + \sum_{k=1}^n \delta_j \Delta_N C_{at-k} + e_t, \quad (5)$$

where $\Delta_N C_{at} = C_{at} - C_{at-N}$ = change in spot price at location a (C_{at}) over time interval k;

$\Delta_N F_t = F_t - F_{t-N}$ = change in futures price (F_t) over time interval k;

$\varepsilon_{1t-N} = C_{at-N} - F_{t-N}$ = lagged value of cash-to-futures basis;

with N = approximate number of days in period of difference; weekly ($N = 5$), monthly ($N = 21$), and quarterly ($N = 63$);

and location $a = 1$ (Alabama), 2 (Chattanooga), 3 (Memphis), 4 (Nashville East), 5 (Nashville West), and 6 (Richmond).

The number of lags, m and n, are determined by the Akaike Criterion and the Ljung-Box test. DF is the degrees of freedom available to estimate the ECM. The T-statistics are constructed using OLS standard errors, since all Ljung-Box tests fail to reject the null hypothesis that the errors are white noise. Price changes are taken between the last days of successive periods.

Panel A. Daily Price Change ($N = 577$)^a

	$\Delta_1 C_1$	$\Delta_1 C_2$	$\Delta_1 C_3$	$\Delta_1 C_4$	$\Delta_1 C_5$	$\Delta_1 C_6$
α_1 t-stat	-.052 (-8.3)***	-.066 (-10.2)***	-.057 (-10.4)***	-.044 (-9.0)***	-.030 (-6.3)***	-.087 (-12.4)***
b_{ECM} t-stat	.030 (2.6)**	.023 (1.9)*	.013 (1.3)	.005 (.5)	.011 (1.2)	-.001 (-.1)
R ² analogue (m, n, DF)	.0048 (2, 2, 568)	.0031 (1, 4, 565)	.00019 (1, 1, 571)	.00039 (1, 4, 565)	.0015 (3, 3, 565)	.000025 (1, 4, 565)
Ljung-Box χ^2_{24} (p-value)	26.7 (.32)	30.9 (.16)	32.2 (.12)	22.8 (.53)	18.3 (.79)	29.5 (.20)

Panel B. Weekly Price Change ($N = 119$)

	$\Delta_5 C_1$	$\Delta_5 C_2$	$\Delta_5 C_3$	$\Delta_5 C_4$	$\Delta_5 C_5$	$\Delta_5 C_6$
α_1 t-stat	-.25 (-5.4)***	-.29 (-5.9)***	-.30 (-5.8)***	-.24 (-5.5)***	-.19 (-5.1)***	-.47 (-7.3)***
b_{ECM} t-stat	.24 (6.3)***	.23 (6.0)***	.23 (5.7)***	.21 (5.3)***	.20 (5.6)***	.26 (6.5)***
R ² analogue (m, n, DF)	.141 (1, 1, 113)	.134 (1, 1, 113)	.129 (1, 1, 113)	.125 (1, 1, 113)	.125 (1, 1, 113)	.151 (1, 3, 109)
Ljung-Box χ^2_{24} (p-value)	24.7 (.42)	20.4 (.68)	28.3 (.25)	23.0 (.52)	27.1 (.30)	30.1 (.18)

Table 6, continued

Panel C. Monthly Price Change (N = 26)

	$\Delta_{21}C_1$	$\Delta_{21}C_2$	$\Delta_{21}C_3$	$\Delta_{21}C_4$	$\Delta_{21}C_5$	$\Delta_{21}C_6$
α_1 t-stat	-.92 (-3.4)***	-.91 (-3.3)***	-1.21 (-4.4)***	-1.14 (-4.2)***	-1.11 (-4.8)***	-1.15 (-3.5)***
b_{ECM} t-stat	.61 (8.2)***	.65 (9.0)***	.59 (8.0)***	.50 (6.3)***	.49 (7.0)***	.73 (10.8)***
R ² analogue (m, n, DF)	.490 (1, 1, 21)	.528 (1, 1, 21)	.528 (2, 1, 19)	.434 (2, 1, 19)	.444 (2, 1, 19)	.555 (1, 1, 21)
Ljung-Box χ^2_6 (p-value)	5.2 (.52)	3.1 (.80)	2.5 (.87)	3.3 (.78)	2.8 (.84)	.6 (.996)

Panel D. Quarterly Price Change (N = 8)^b

	$\Delta_{63}C_1$	$\Delta_{63}C_2$	$\Delta_{63}C_3$	$\Delta_{63}C_4$	$\Delta_{63}C_5$	$\Delta_{63}C_6$
α_1 t-stat	-2.18 (-2.5)**	-2.27 (-2.3)**	-2.10 (-3.0)***	-1.71 (-2.7)**	-1.57 (-2.2)**	-2.34 (-1.5)
b_{ECM} t-stat	1.01 (6.5)***	1.04 (6.5)***	.98 (9.7)***	.88 (8.1)***	.84 (6.2)***	1.08 (6.6)***
R ² analogue (m, n, DF)	.654 (1, 1, 3)	.650 (1, 1, 3)	.780 (1, 1, 3)	.712 (1, 1, 3)	.647 (1, 1, 3)	.843 (1, 1, 3)
Ljung-Box χ^2 (p-value)	- -	- -	- -	- -	- -	- -

* indicates statistical significance at the .10 level; ** at the .05 level; and *** at the .01 level.

^a Note that the results of estimating the ECM on daily changes in Panel A of Table 6 differ slightly from the analogous results presented in Table 3. These differences are a result of: (i) replacing the error correction term, ϵ_{1-t-N} , with the futures-to-cash basis ($C_{at-N} - F_{t-N}$), and (ii) including the contemporaneous change in the futures price (ΔF_t) in the ECM for Table 6.

^b Note that “-” indicates insufficient data are available to conduct the Ljung-Box test in Panel D. Analogous tests are not conducted for six-month price changes, since there are only three non-overlapping six-month intervals during our sample period, and thus insufficient degrees of freedom to estimate the ECM.