# Technical Trading: A Trend Factor

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# Technical Trading: A Trend Factor


**Abstract**

In this paper, we provide a general equilibrium model to analyze how technical traders compete trading with informed investors and how they affect stock prices. Based on the model, we propose a trend factor that captures cross-section short-, intermediate- and long-term stock price trends. The trend factor has an average return of 1.63% per month, doubling the market average return and quadrupling the market Sharpe ratio. During the recent financial crisis, it earns 0.79% per month while the market loses −2.03% per month. From an asset pricing perspective, it helps in explaining cross-section stock returns. Internationally, the trend factor also performs well in other G7 countries.



*JEL Classification:* G11, G14

*Keywords:* Trends, Moving Averages, Asymmetric Information, Predictability, Momentum, Factor Models.

# I Introduction

In practice, many top traders and successful fund managers, who call themselves technical or non-discretionary traders, use moving averages (MAs) of past prices to learn about the market trends and make their investment decisions accordingly (see, e.g., Schwager (1989; 2012) and Narang (2013)). Fung and Hsieh (2001) find that trend-following is of great importance for explaining hedge fund returns. Burghardt and Walls (2011) show that a simple mechanical trading rule based on the MAs can yield favorable returns in trading futures contracts and the return correlation with the returns on the managed futures index exceeds 70%.[1] Olszweski and Zhou (2014) demonstrate further that a suitable technical trading strategy with the MAs can yield comparable performance to an independent strategy based on economic fundamentals. In the academic literature, Brock, Lakonishok, and LeBaron (1992) seem the first major study that provides convincing evidence on the predictive power of the MAs.[2] Lo, Mamaysky, and Wang (2000) strengthen further the evidence with automated pattern recognition analysis. Recently, Neely, Rapach, Tu, and Zhou (2014) find that technical indicators, primarily the MAs, have forecasting power on the stock market matching or exceeding that of macroeconomic variables. Theoretically, Zhu and Zhou (2009) show that trading based on the MAs is a rational strategy that can outperform an asset pricing model-dependent strategy under model or parameter uncertainty. But their analysis is in a partial equilibrium framework that assumes zero impact of technical trading on stock prices.

In this paper, we make two contributions to technical trading using the MAs. The first is to propose a continuous-time general equilibrium model with a hierarchical information structure. There are three types of investors, the informed investors, the technical and the noise traders. While the informed investors know more of the innovations of the economic fundamentals, the technical traders only observe the dividends and prices, and they utilize the MA of historical prices to help them making trading decisions. The noise traders simply have inelastic demand for liquidity, and hence the price in the model is not fully revealing. In equilibrium, as will be shown later, the informed investors can extract more "rents" from providing liquidity to noise traders than in an economy without the technical traders.

[1] Cao, Rapach, and Zhou (2013) find that the managed futures, along with the Global Macro whose goal is to identify and profit from global macroeconomic trends, are the top hedge fund styles that perform well in both good and bad times of the market.

[2] Cowles (1933) provides perhaps the first academic study on the MA or technical analysis, and Fama and Blume (1966) examine more filter rules. But results of the earlier studies are mixed and inconclusive.

However, the technical traders, even with inferior information, can survive and earn an equilibrium return for providing a risk-sharing function to the market. As a result, the MA signal used by the technical traders is a priced risk factor which depends on the population ratio of the technical traders in the market. When the ratio is small, the technical traders will behave as trend-followers. However, when the ratio is large, they can be contrarians.

Our model has two interesting empirical implications. First, it provides a theoretical basis for using the moving averages in an explicit functional form, i.e., the average price divided by the current price. This functional form not only states that the moving average price is useful for forecasting the market return, but also provides a way of how to do it explicitly. In contrast, the vast empirical studies on the MAs only apply them as an indicator function to indicate an up or down state of the asset price. Second, our model offers a theoretical explanation of the time series momentum (TSMOM) newly documented by Moskowitz, Ooi, and Pedersen (2012). They find that an asset's own past 12 month return predicts its future return in the next month across asset classes (country equity indices, currencies, commodities, and bonds). Although TSMOM is not explained by existing theories, it can be explained by our model. Assume that the technical investors learn from the markets by simply using MAs of a lag length $L = 12$ months. Then, when they trade with informed traders, their collective price impact will not be arbitraged away, and hence the equilibrium price will allow for a short-term positive return correlation within 12 months, and a long-term negative correlation beyond.

Our paper is related to the theoretical literature on investment under informational inefficiency. Rational models, such as Treynor and Ferguson (1985), Brown and Jennings (1989), Brock and Hommes (1998), Griffioen (2003), Chiarella, He, and Hommes (2006), and Cespa and Vives (2012), show that past prices are useful for forecasting future prices under informational inefficiency. Brunnermeier (2001), Chamley (2003) and Vives (2008) summarize many additional models. Moreover, from the behavioral finance perspective, De Long, Shleifer, Summers, and Waldmann (1990) and Hong and Stein (1999) argue that behavior biases can lead to price trends. However, there are no equilibrium models that link the MAs explicitly to expected stock returns except ours. Mathematically, our model follows closely Wang (1993) who provides the first dynamic asset pricing model under asymmetric information with a closed-form solution. Unlike his paper, we have technical traders who use the MAs to update their information instead of an optimal filter.[3] We also obtain a closed-form solution to our model, which is, to our knowledge, the first general equilibrium model that analyzes explicitly the role of technical trading in the financial markets by using the MAs of past prices.

The second contribution of this paper is empirical. Based on the MA signal from our theoretical model, we provide a single trend factor that synthesizes the short-, intermediate- and long-term price trends of the stock markets. While it is well documented in the literature that stock prices have trends across different horizons, they are analyzed separately. For example, Lehmann (1990), Lo and MacKinlay (1990), and Jegadeesh (1990) document the short-term reversals (at daily, weekly, and monthly level, respectively), Jegadeesh and Titman (1993) study the momentum effects (6–12 month price continuation), and DeBondt and Thaler (1985) focus on the long-term reversal (3–5 years). In contrast, our methodology utilizes all information from the short- to the long-run, and hence our paper may be viewed as providing a unified framework for the three known trends: the short-term reversal, the momentum and the long-term reversal.

In constructing the trend factor, we run monthly cross-section regressions of stock returns on their MAs of past prices from different horizons ranging from three days to as many as 1000 days (roughly four trading years), and then the expected returns of next month are estimated. Similar to the construction of most risk factors, we buy those stocks with the highest forecasted expected returns and short those with the lowest. The spread portfolio is our trend factor. Empirically, we find that, with data from June 1930 to December 2013, the trend factor earns an average return of 1.63% per month, and a risk-adjusted abnormal return about 1.56% per month (out-of-sample).[4] In contrast, the short-term reversal, momentum, and long-term reversal factors earn only 0.70%, 0.65% and 0.35% per month, respectively, less than half of what the trend factor produces. The Fama-French size and book-to-market factors earn even less, 0.28% and 0.41% per month, respectively. In terms of the Sharpe ratio, the trend factor has a monthly value of 0.47, more than double those of the short-term reversal, momentum and long-term reversal factors, and more than quadruple those of the

[3] An alternative for updating the information is to use higher order expectations, on which Hellwig (1980) and Singleton (1987) provide some of the earlier major results, and Makarov and Rytchkov (2012) deliver some of the latest advances. Although this line of models can explain certain price patterns such as serial correlation, they are generally extremely difficult to solve, and especially so with the MAs.

[4] The main results on the trend factor have been independently reproduced by many practitioners, PhDs and some professors.

Fama-French three factors. Moreover, the trend factor earns 0.79% per month during the recent financial crisis, while the market and the momentum suffer −2.03% and −1.33% per month, respectively.

Since the trend factor relies on stock price trends from the short- to the long-run, an important question is whether it can be replicated by a suitable portfolio of the short-term reversal, momentum, and long-term reversal factors. If this were the case, running the cross-section regressions at the stock level to forecast expected returns would be totally unnecessary. To answer this question, we carry out six different mean-variance spanning tests (see, e.g., Kan and Zhou, 2012, for a complete review), and find that the tests reject strongly the hypothesis that a portfolio of the three factors can yield a factor that is close to the trend factor in terms of the Sharpe ratio. In other words, the cross-section regressions based on our theory of technical trading at the stock level do capture valuable information beyond that of the short-term reversal, momentum, and long-term reversal factors.

From an asset pricing perspective, the trend factor seems to perform better than the widely used momentum factor in explaining the cross-section portfolio returns. For standard portfolios such as those sorted by the short-term reversal, the trend factor has smaller aggregate pricing error than the momentum factor. For portfolios sorted by the short-term reversal, momentum, and long-term reversal, the aggregate pricing error of the trend factor is again smaller than that of the momentum factor.

The abnormal returns on the trend factor are robust to alternative formations and to alternative data sets. Regardless of whether or not imposing the price and/or size filters on stocks or using a different way of constructing the spread portfolio, the alternatively formed trend factors have similar or even larger abnormal returns. Internationally, the trend factor yields significantly positive abnormal returns for all the other G7 countries (G6) from the available sample period January, 1990 to December, 2013, whereas the short-term reversal, momentum, and long-term reversal factors are insignificant in several G6 countries. While the trend factor does not outperform the momentum factor in some of the G6 countries due to weak reversals, it does outperform it in all G6 countries during the recent financial crisis.

The paper proceeds as follows. Section II lays out the assumptions of the model. Section III presents the main theoretical results. Section IV analyzes the equilibrium properties of the model. Section V provides the trend factor. Section VI and VII investigate its robustness and pricing ability. Section VIII concludes.

# II The Model

Following Wang (1993), we consider an economy with a single risky asset which pays out its earnings as a random stream of dividends. There is a market for the asset which is traded as an infinitely divisible security and the market is populated with two types of traders: the well-informed rational investors and less informed technical traders who trade based on the MA signal (together with other public signals). In addition, there are noise traders whose demand is modeled by an exogenous fluctuating total amount of the asset. Technically, noise trader's demand might be endogenously modeled, but the additional complexity would not alter our main results.

The information structure is hierarchical, that is, the information set of the informed investors encompasses that of the technical traders. The model is set in continuous-time with infinite horizon. Wang (1993) uses a very similar model to study the impact of information asymmetries on the time series properties of prices, such as the risk premiums, price volatility and the negative autocorrelation in returns. In contrast, our model, featuring the technical traders, aims at explaining the use of the MA signal and the short-run positive and long-run negative return correlations.

Before pursuing the model any further, we address first two common motivational questions. The first question is what the theoretical reasons are that explain the use of the MAs in the real world. If the stock market is a pure random walk, it is clear that the MAs will be useless for trading stocks as they simply add noise. Hence, the success of the MAs depends on price trends as they are the simple econometric methods that capture the trends. Theoretically, there are at least four types of models that justify price trends in the market. First, when investors do not receive information at the same time or heterogeneously informed, Treynor and Ferguson (1985) and Brown and Jennings (1989) demonstrate that past prices enable investors to make better inferences about future prices, and Grundy and McNichols (1989) and Blume, Easley, and O'Hara (1994) show that trading volume can provide useful information beyond prices. Second, if there is asset residual payoff uncertainty and/or persistence in liquidity trading, Cespa and Vives (2012) show that asset prices can deviate from their fundamental values and rational long-term investors follow trends. Third, due to behavioral biases, Hong and Stein (1999) explain that, at the start of a trend, investors underreact to news; as the market rises, investors subsequently overreact, leading to even higher prices. Fourth, with the presence of positive feedback traders (who buy after prices rise and sell after prices fall as emphasized by the hedge fund guru Soros (2003)), De Long et al. (1990), and Edmans, Goldstein, and Jiang (2012) show that prices can trend rationally. Empirically, Moskowitz et al. (2012) find that pervasive price trends exist across commonly traded equity indices, currency, commodity, and bond futures. Therefore, as long as the stock market is not a pure random walk and exhibits periodic trends, the MA signal should be a useful investment tool.[5]

The second question is why most investors in practice use the MAs instead of a more sophisticated time series model to capture price trends? The moving averages are simple and easy to use. As put by the technical guru Murphy (1999) in his famous book, *Technical Analysis on the Financial Markets*, "Moving average is one of the most versatile and widely used of all technical indicators. Because of the way it is constructed and the fact that it can be so easily quantified and tested, it is the basis for many mechanical trend-following systems in use today...... Chart analysis is largely subjective and difficult to test. As a result, chart analysis does not lend itself that well to computerization. Moving average rules, by contrast, can easily be programmed into a computer, which then generates specific buy and sell signals." From the behavioral finance point of view, people at large prefer simple rules to complex ones. However, the use of the MAs seems quite rational too. A time series model such as an autoregressive process requires a large amount of stationary data to estimate accurately, but the real world data are non-stationary with changing regimes and parameters. In fact, it is often the case that complex models underperform simple models out-of-sample. For example, DeMiguel, Garlappi, and Uppal (2007) show a simple equal-weighting portfolio rule beats more sophisticated strategies; Timmermann (2006) and Rapach, Strauss, and Zhou (2010) find that a simple average forecast rule provides better forecasts than from more complex econometric models. More closely related, Zhu and Zhou (2009) show that the MA is a robust learning approach compared with sophisticated ones. Hence, the technical traders in our model are assumed to make their investment decisions using the MA.

[5]Learning from market past prices is valuable both theoretically and empirically not only for investors, but also for governments (see, e.g., Bernanke and Woodford, 1997; Bond and Goldstein, 2012, and references therein.)

Formally, we make below the same assumptions as Wang (1993), except Assumption 5 which specifies the MA as the learning tool for the technical traders instead of an optimal filter.

*Assumption* 1. The market is endowed with a certain amount of one risky asset, each unit of which provides a dividend flow given by

$$dD_t = (\pi_t - \alpha_D D_t)dt + \sigma_D dB_{1t}, \tag{1}$$

where $\pi_t$ is the mean level of dividend flow given by another stochastic process

$$d\pi_t = \alpha_\pi(\bar{\pi} - \pi_t)dt + \sigma_\pi dB_{2t}, \tag{2}$$

where $B_{1t}$ and $B_{2t}$ are independent innovations.

The mean-reverting Gaussian processes together with the CARA utility specified below are known to yield closed-form solutions and hence a workhorse model in the study of financial markets with asymmetric information (see, e.g., Vives (2008)). When $\alpha_D > 0$, $\pi_t/\alpha_D$ is the short-run steady-state level of the dividends. The mean-reversion in $\pi_t$ allows for business cycles in the economy.

*Assumption* 2. The supply of the risky asset is $1 + \theta_t$ with

$$d\theta_t = -\alpha_\theta \theta_t dt + \sigma_\theta dB_{3t}, \tag{3}$$

where $B_{3t}$ is another Brownian Motion independent of both $B_{1t}$ and $B_{2t}$. Assumption 2 normalizes the long-run stationary level of the supply of the risky asset to 1, whereas $\theta_t$ represents shocks away from that level which implicitly allows liquidity trades outside the model. The fluctuating supply of risky asset is exogenously given so the supply is inelastic, which resembles a demand schedule by noise traders.

*Assumption* 3. The claim on the risky asset is infinitely divisible and shares are held by the investors in the economy. Shares are traded in a competitive stock market with no transaction cost. The stock is the only security traded in the market. Let $P$ be the equilibrium price of the stock.

*Assumption* 4. There is a risk-free investment to all investors with a constant rate of return $1 + r$ ($r > 0$). This can be a risk free bond with perfect elastic supply.

*Assumption* 5. Apart from noise traders, there are two types of investors: the informed and the technical. Let $w$ be the fraction of the technical traders. The informed investors observe the dividend $D_t$, the mean growth rate of dividend $\pi_t$, the price as well as all history of the variables, while they do not directly observe the supply of asset. Formally, $\mathcal{F}^i(t) = \{D_\tau, P_\tau, \pi_\tau : \tau \leq t\}$ is the informed investors' information set at time $t$. On the other hand, technical traders only observe the dividend and price, and do not directly observe the mean grow rates of the dividends. Instead of optimally learning the unobservables, we assume that the technical traders infer information from the historical prices via the MA, which is defined in our model as an exponentially weighted average of the past prices,

$$A_t \equiv \int_{-\infty}^{t} \exp\left[-\alpha(t-s)\right] P_s ds, \tag{4}$$

with $\alpha > 0$. We use the exponentially weighted moving average rather than a simple arithmetic moving average since it is theoretically convenient in obtaining a close-form solution. The parameter $\alpha$ controls the size of moving average window. Note that

$$dA_t = (P_t - \alpha A_t)dt, \tag{5}$$

which is tractable for analytical analysis. Formally, $\mathcal{F}^u(t) = \{1, D_t, P_t, A_t\}$ is the information set of the technical traders at time $t$. Note that the technical traders are assumed to use the MA as their summary statistic for historical information, the information process is Markovian.

*Assumption* 6. The structure of the market is common knowledge.

*Assumption* 7. The investors have expected additive utility, $E[\int u(c(\tau), \tau)d\tau|\cdot]$, conditional on their respective information set, with constant absolute risk aversion (CARA)

$$u(c(t), t) = -e^{-\rho t - c(t)}, \tag{6}$$

where $\rho$ is the discount parameter, and $c(t)$ is the consumption rate at time $t$.

# III Equilibrium

In this section, we solve for the equilibrium of the economy defined in the previous section. For easy understanding, we summarize the main theoretical results and their implications first, and then provide the solution in detail.

## A Main Results

The equilibrium concept is that of rational expectations developed by Lucas (1978), Green (1973), Grossman (1976), among others. Even though there is a certain bounded rationality due to cost to optimal learning, the model can still be viewed as a rational one because there is no resort to the use of any individual psychological bias.

The main theoretical result is the following:

**Theorem 1:** *In an economy defined in Assumptions 1-7, there exists a stationary rational expectations equilibrium. The equilibrium price function has the following linear form:*

$$P_t = p_0 + p_1 D_t + p_2 \pi_t + p_3 \theta_t + p_4 A_t, \tag{7}$$

*where* $p_0, p_1, p_2, p_3$ *and* $p_4$ *are constants determined only by the model parameters.*

The theorem says that the equilibrium price is a linear function of the state variables and the MA signal $A_t$. In contrast to a standard model without technical traders, $A_t$ is the new priced factor in the market. Economically, it is clear that $p_0, p_1$ and $p_2$ are all positive, reflecting the positive price impact of the state variables. However, $p_4$ can be either positive or negative, which depends primarily on the proportion of technical traders in the market.

A major implication of our model is that the stock price can be predicted by the MA signal. Specifically, taking finite difference in Equation (7) with discrete time interval $\Delta t$, and using Equation (5), we obtain

$$\Delta P_t = p_1 \Delta D_t + p_2 \Delta \pi_t + p_3 \Delta \theta_t + p_4 (P_t - \alpha A_t) \Delta t.$$

After dividing by price, we have the following predictive regression on the moving average,

$$r_{t+1} = a + \beta \frac{A_t}{P_t} + \epsilon_t, \tag{8}$$

where the impact of other variables are summarized in the noise term of the regression. In empirical applications, $A_t$ can be approximated by the simple moving average

$$A_t = \frac{1}{L} \sum_{i=0}^{L-1} P_{t - i\Delta t}, \tag{9}$$

where $L$ is lag length or the moving average window. Theoretically, our model states that $A_t / P_t$ should be a predictor of the asset return. If the model is applied to asset classes, similar conclusions hold. However, in the real world, the proportion of technical traders can change over time, and hence the slope on $A_t/P_t$ may not always be positive or negative.

Another implication of our model is to provide perhaps the first theoretical explanation of the TSMOM. To illustrate this, we need to show that our model allows for a short-term positive return correlation at a lag of $L$ about one year and a long-term negative return correlation beyond $L$. Mathematically, it suffices to show that the return autocovariance has such a pattern. Indeed, the autocovariance of the return in our model is

$$\begin{aligned}\frac{1}{\tau^2} < P_{t+\tau} - P_t, P_t - P_{t-\tau} > &= p_1^2[A_D g(\alpha_D, \tau) + B_D g(\alpha_1, \tau)] \\ &+ p_2^2[A_\pi g(\alpha_\pi, \tau) + B_\pi g(\alpha_1, \tau)] \\ &+ p_3^2[A_\theta g(\alpha_\theta, \tau) + B_\theta g(\alpha_1, \tau)], \end{aligned} \tag{10}$$

where the parameters $A_D, B_D, A_\pi, B_\pi, A_\theta, B_\theta$ are derived in Appendix F, and

$$g(\alpha, \tau) = -\frac{(1 - e^{-\alpha\tau})^2}{\tau^2}.$$

Figure 1 provides a numerical example. The autocovariance is positive up till $\tau = 1$ year, and then become negative for longer horizons, explaining the short-term positive and long-term negative correlation of the TSMOM. In contrast, for the most promising fully rational model in this regard, Wang (1993) can generate either positive or negative autocovariance for some given set of parameters, but cannot generate both short-run positive and long-run negative autocovariance for the same set of parameters.

The proof of Theorem 1 can be derived based on the optimal stock demand functions of both informed and technical investors. This is because Equation (7) implies

$$\begin{aligned} dA_t &= (P_t - \alpha A_t)dt \\ &= [p_0 + p_1 D_t + p_2 \pi_t + p_3 \theta_t + (p_4 - \alpha) A_t]\, dt. \end{aligned} \tag{11}$$

Due to the stationarity of $D_t, \pi_t$ and $\theta_t$, it is easy to show that a stationary solution exists when $p_4 < \alpha$. Then, with the help of the explicit solutions to the demand functions, Theorem 1 follows from the market clearing condition, which are the subjects of the following subsections.

## B Informed Investors

We first describe the information set and the investment opportunity for the informed investors, which are given by the following lemma:

**Lemma 1:** For informed investors the information set is given by a vector of state variables $\Psi^i = (1, D_t, \pi_t, \theta_t, A_t)^T$, which satisfies the following SDE:

$$d\Psi^i = e^i_\Psi \Psi^i dt + \sigma^i_\Psi dB^i_t, \tag{12}$$

where $B^i_t$ is a 5-dimensional Brownian Motion, and $e^i_\Psi, \sigma^i_\Psi \in R^{5\times 5}$, constant matrices, all of which are defined in Appendix A. The investment opportunity satisfies the stochastic differential equation:

$$dQ = (D - rP)dt + dP = e^i_Q \Psi dt + \sigma^i_Q dB, \tag{13}$$

with $e^i_Q \in R^{5\times 1}$ and $\sigma^i_Q \in R^{5\times 1}$ which are given in Appendix A.
Proof. See Appendix A.

Then, the informed investors' optimization problem is

$$\max_{\eta^i, c^i} E\left[-\int_t^\infty e^{-\rho s - c(s)} ds | \mathcal{F}^i_t\right] \; s.t. \; dW^i = (rW^i - c^i)dt + \eta^i dQ. \tag{14}$$

Let $J^i(W^i, D_t, \pi_t, \theta_t, A_t; t)$ be the value function, which satisfies the HJB equation

$$\begin{aligned} 0 \;=\; \max_{c,\eta} \Bigg[ & -e^{-\rho t - c} + J_W(rW^i - c^i + \eta^i e^i_Q \Psi) + \frac{1}{2}\sigma^i_Q \sigma^{iT}_Q \eta^{i2} J_{WW} + \eta^i \sigma^i_Q \sigma^{iT}_\Psi J_{W\Psi} \\ & -\rho J + (e^i_\Psi \Psi)^T J_\Psi + \frac{1}{2}\sigma^i_\Psi J_{\Psi\Psi} \sigma^{iT}_\Psi \Bigg]. \end{aligned} \tag{15}$$

The solution to the optimization problem is provided by
**Proposition 1:** Equation (15) has a solution of the form:

$$J^i(W^i, D_t, \pi_t, \theta_t, A_t; t) = -e^{-\rho t - rW - \frac{1}{2}\Psi^{iT} V^i \Psi^i}, \tag{16}$$

with $\Psi^i = (1, D_t, \pi_t, \theta_t, A_t)^T$, and $V^i \in R^{5\times 5}$, a positive definite symmetric matrix. The optimal demand for the stock is given by

$$\eta^i = f^i \Psi^i = f^i_0 + f^i_1 D_t + f^i_2 \pi_t + f^i_3 \theta_t + f^i_4 A_t, \tag{17}$$

where $f^i_0, f^i_1, f^i_2, f^i_3$ and $f^i_4$ are constants.
Proof. See Appendix B.

Proposition 1 says that, given the model assumptions, the informed investors' demand for the stock is a linear function of the fundamental variables, $D_t, \pi_t, \theta_t$, as well as the technical variable $A_t$.

## C Technical Traders

Technical traders are in a different situation from the informed investors in that they face different information set and different perceived investment opportunity. In particular, they do not observe state variable $\pi_t$ as the informed investors do. However, they do know the dynamics of the processes. In our setting, they use the MA signal of past stock prices together with the other observables $(D_t, P_t)$ to infer $\pi_t$. That is, they learn about $\pi_t$ from its projection onto their information set $\Psi^u = (1, D_t, P_t, A_t)$. Specifically, they use the following linear regression to infer $\pi_t$, denoted as $\pi_t^u$,

$$\pi_t^u = \beta_0 + \beta_1 D_t + \beta_2 P_t + \beta_3 A_t + \sigma_u u_t. \tag{18}$$

This is the optimal estimation for the unobservable $\pi_t$ given the information set $\Psi^u$. With the insights from Wang (1993), we assume that the technical traders, who do not observe $\pi_t$, know the process that drives its dynamics, and hence they can infer the unconditional linear regression coefficients in Equation (18).[6] The technical details for computing the parameters $\beta_0, \beta_1, \beta_2$, $\beta_3$ and $\sigma_u$ are given in Appendix C.

Given the price in Equation (7), the technical traders infer their estimation of the state variable $\theta_t$, $\theta_t^u$, from the price via

$$\begin{aligned} \theta_t^u &= \frac{1}{p_3}[P_t - (p_0 + p_1 D_t + p_2 \pi_t^u + p_4 A_t)] \\ &= \gamma_0 + \gamma_1 D_t + \gamma_2 P_t + \gamma_3 A_t - \frac{p_2}{p_3}\sigma_u u_t, \end{aligned} \tag{19}$$

where the parameters $\gamma_0, \gamma_1, \gamma_2$ and $\gamma_3$ are given in Appendix C.

Define

$$\hat{\pi}_t = \beta_0 + \beta_1 D_t + \beta_2 P_t + \beta_3 A_t, \tag{20}$$

$$\hat{\theta}_t = \gamma_0 + \gamma_1 D_t + \gamma_2 P_t + \gamma_3 A_t. \tag{21}$$

[6]For models of this type, Brock and Hommes (1998) use another strategy by assuming no common knowledge on the underlying dynamics while investors choose among different predicting rules, where chaotic price dynamics may emerge. Here our assumption enables us to focus on characterizing the MA rule in a stationary equilibrium.

The dynamics of $D_t$ for the technical traders are then

$$\begin{aligned} dD_t &= (\hat{\pi}_t + \sigma_u u_t - \alpha_D D_t)dt + \sigma_D dB_{1t} \\ &= (\hat{\pi}_t - \alpha_D D_t)dt + \sigma_D dB_{1t} + \sigma_u dZ_t \\ &= (\hat{\pi}_t - \alpha_D D_t)dt + \hat{\sigma}_D dB^u_{1t}, \end{aligned} \tag{22}$$

where $Z_t$ is defined as $Z_t = \int_0^t u_s ds$, which is another independent Brownian motion with $u_t$ the white noise in regression equation (18). The third equality in Equation (22) has used

$$\hat{\sigma}_D dB^u_{1t} = \sigma_D dB_{1t} + \sigma_u dZ_t, \tag{23}$$

with

$$\hat{\sigma}^2_D = \sigma^2_D + \sigma^2_u. \tag{24}$$

Defining another state variable,

$$\Lambda_t = p_2 \pi_t + p_3 \theta_t, \tag{25}$$

which is observable by technical traders through observing the equilibrium price and dividend, we have then

$$\begin{aligned} d\Lambda_t &= (p_2 \alpha_\pi (\bar{\pi} - \hat{\pi}_t) - p_3 \alpha_\theta \hat{\theta}_t)dt \\ &\quad + p_2(\sigma_\pi dB_2 - \alpha_\pi \sigma_u dZ_t) + p_3(\sigma_\theta dB_3 + \alpha_\theta \frac{p_2}{p_3} \sigma_u dZ_t) \\ &= (p_2 \alpha_\pi (\bar{\pi} - \hat{\pi}_t) - p_3 \alpha_\theta \hat{\theta}_t)dt + \hat{\sigma}_\Lambda dB^u_{2t}, \end{aligned} \tag{26}$$

with

$$\begin{aligned} \hat{\sigma}_\Lambda dB^u_{2t} &= p_2(\sigma_\pi dB_{2t} - \alpha_\pi \sigma_u dZ_t) + p_3(\sigma_\theta dB_{3t} + \alpha_\theta \frac{p_2}{p_3} \sigma_u dZ_t) \\ &= p_2 \sigma_\pi dB_{2t} + p_3 \sigma_\theta dB_{3t} + (\alpha_\theta - \alpha_\pi) p_2 \sigma_u dZ_t \end{aligned} \tag{27}$$

and

$$\hat{\sigma}^2_\Lambda = (p_2 \sigma_\pi)^2 + (p_3 \sigma_\theta)^2 + (\alpha_\theta - \alpha_\pi)^2 p_2^2 \sigma_u^2. \tag{28}$$

Based on Equations (23) and (27), the correlation between $dB^u_{1t}$ and $dB^u_{2t}$, defined as

$$\mathrm{Var}(dB^u_{1t}, dB^u_{2t}) \equiv \varrho dt,$$

can be written as

$$\varrho = \frac{p_2 \sigma_u^2 (\alpha_\theta - \alpha_\pi)}{\hat{\sigma}_D \hat{\sigma}_\Lambda}. \tag{29}$$

With the above discussions, we can summarize the investment opportunity faced by the technical traders as in the following:

**Lemma 2:** The state variable set observed by technical traders, $\Psi^u = (1, D_t, P_t, A_t)^T$, follows a stochastic differential equation

$$d\Psi^u = e^u_\psi \Psi^u dt + \sigma^u_\Psi dB^u_t, \tag{30}$$

where $B^u_t = (0, B^u_{1t}, B^u_{2t}, 0)$, with $B^u_{1t}$ and $B^u_{2t}$ defined in Equations (23) and (27), and $e^u_\Psi$ and $\sigma^u_\Psi$ given by

$$e^u_\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \beta_0 & \beta_1 - \alpha_D & \beta_2 & \beta_3 \\ q_0 & q_1 & q_2 & q_3 \\ 0 & 0 & 1 & -\alpha \end{pmatrix}, \quad \sigma^u_\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \hat{\sigma}_D & 0 & 0 \\ 0 & p_1\hat{\sigma}_D + \varrho\hat{\sigma}_\Lambda & \sqrt{1-\varrho^2}\hat{\sigma}_\Lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{31}$$

The investment opportunity is

$$\begin{aligned} dQ &= (D - rP)dt + dP \\ &= e^u_Q \Psi^u dt + \sigma^u_Q dB^u_t, \end{aligned}$$

with $e^u_Q$ and $\sigma^u_Q$ defined as

$$e^u_Q = \begin{pmatrix} q_0 & 1+q_1 & q_2 - r & q_3 \end{pmatrix}, \quad \sigma^u_Q = \begin{pmatrix} 0 & p_1\hat{\sigma}_D + \varrho\hat{\sigma}_\Lambda & \sqrt{1-\varrho^2}\hat{\sigma}_\Lambda & 0 \end{pmatrix}. \tag{32}$$

Proof: See Appendix D.

To solve the technical traders' optimization problem, let $W^u$ be the wealth of a technical trader, $\eta^u$ be the holding of the stock and $c^u$ be the consumption. Then the optimization problem is

$$\max_{\eta^u, c^u} E\left[-\int_t^\infty e^{-\rho s - c(s)} ds | \mathcal{F}^u_t\right] \;\; s.t. \;\; dW = (rW^u - c^u)dt + \eta^u dQ. \tag{33}$$

Let $J^u(W^u, \Psi^u; t)$ be the value function, which satisfies the following HJB equation,

$$\begin{aligned} 0 \;=\; \max_{c^u,\eta^u} &\Big[-e^{-\rho t - c^u} + J^u_{W^u}(rW^u - c^u + \eta^u e^u_Q \Psi^u) + \frac{1}{2}\sigma^u_Q \sigma^{uT}_Q \eta^{u2} J_{W^u W^u} + \eta^u \sigma^u_Q \sigma^{uT}_\Psi J^u_{W^u \Psi^u} \\ &-\rho J^u + (e^u_\Psi \Psi^u)^T J^u_\Psi + \frac{1}{2}\sigma^u_\Psi J^u_{\Psi^u\Psi^u} \sigma^{uT}_\Psi\Big]. \end{aligned} \tag{34}$$

The solution is provided by the following:

**Proposition 2:** Equation (34) has a solution of the following form,

$$J^u(W^u, D_t, P_t, A_t; t) = -e^{-\rho t - rW^u - \frac{1}{2}\Psi^{uT} V^u \Psi^u}, \tag{35}$$

with

$$\Psi^u = (1, D_t, P_t, A_t)^T, \tag{36}$$

and $V^u \in R^{4\times4}$, a positive definite symmetric matrix. The technical traders' optimal demand for the stock is given by

$$\eta^u = f_0^u + f_1^u D_t + f_2^u P_t + f_3^i A_t, \tag{37}$$

where $f_0^u, f_1^u, f_2^u$ and $f_3^i$ are constants.
Proof. See Appendix E.

Proposition 2 says that the technical traders' demand for stock is a linear function of state variables, $D_t, P_t$, observable to them, and $A_t$, the technical indicator they use. Note that, in contrast, the price $P_t$ is not in the demand function of the informed investors because, to them, $\theta_t$ and $P_t$ are equivalent in terms of information content. On the other hand, technical traders observe neither $\pi_t$ nor $\theta_t$, and hence they can only pin down their demand through the price function $P_t$. Indeed, to them, $P_t$ provides new information.

## D Market Clearing

Given Equations (17) and (37), the demands of stock by informed and technical investors, the market clearing condition requires

$$\eta^i + \eta^u = 1 + \theta_t,$$

or equivalently,

$$(1-w)[f_0^i + f_1^i D_t + f_2^i \pi_t + f_3^i \theta_t + f_4^i A_t] + w[f_0^u + f_1^u D_t + f_2^u P_t + f_3^u A_t] = 1 + \theta_t. \tag{38}$$

Substitute $P_t$ in (7) into above, and by matching coefficients of state variables, we obtain

$$\begin{aligned}
(1-w)f_0^i + w(f_0^u + f_2^u p_0) &= 1, \\
(1-w)f_1^i + w(f_1^u + f_2^u p_1) &= 0, \\
(1-w)f_2^i + w(f_2^u p_2) &= 0, \\
(1-w)f_3^i + w(f_2^u p_3) &= 1, \\
(1-w)f_4^i + w(f_3^u + f_2^u p_4) &= 0.
\end{aligned} \tag{39}$$

The solution to Equation (39) determines the coefficients $p_0, p_1, p_2, p_3$ and $p_4$ for the price function of (7). It is easy to show that a solution exists under general conditions. This implies that Theorem 1 holds.

If all the investors are informed, i.e., $w = 0$, the four constant parameters in Equation (7) can be solved explicitly. In general, their associated Ricatti algebraic equations can be easily solved by using the Matlab function CARE, and then the solution to equations (39) can be solved via any nonlinear least square solver.

# IV Price and Trading

In this section, we characterize the equilibrium price in our model and examine the impact of information structure and the trading strategy.[7]

## A Stock Price and Information Structure

In our model, the parameter $w$ characterizes the information structure of the economy. By changing $w$, we can see how information structure can impact on the prices. The base case parameter is set at

$$r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.6, \sigma_\theta = 3.0, \alpha_\pi = 0.2, \alpha_\theta = 0.4, \alpha_D = 1.0.$$

To better characterize the solution, we re-arrange Equation (7) as

$$P_t = p + p_D D_t + p_\pi \pi_t + p_\theta \theta_t + p_{mv} MV_t, \tag{40}$$

where

$$MV_t \equiv P_t - \alpha A_t, \tag{41}$$

and

$$p = \frac{p_0}{1 - p_{mv}}, \; p_D = \frac{p_1}{1 - p_{mv}}, \; p_\pi = \frac{p_2}{1 - p_{mv}}, \; p_\theta = \frac{p_3}{1 - p_{mv}}, \; p_{mv} = \frac{p_4}{p_4 - \alpha}. \tag{42}$$

There are two advantages for doing so. First, in the stationary equilibrium, the last term of Equation (40) has mean 0. Second, that $p_{mv} < 1$ is a necessary and sufficient condition for $P_t$ in Equation (40) to be stationary. When $p_{mv} > 0$, the price process in (40) indicates that the MA rule exhibits a trend-following type of behavior; while when $p_{mv} < 0$, the MA signal is used for reverse trading. We will show that both price dynamics are possible for different parameterizations.

[7] We have also conducted a comparative statics analysis and reported the results in an online appendix.

Table I presents the numerical solution, examining price impact of information structure. Panel A of Table I shows the (re-arranged) equilibrium price parameters in Equation (40) with the base case parameters for $\alpha = 1$. The price sensitivity to $D_t$, measured by $p_D$, increases from 0.9526 to 2.3489 when the ratio of technical traders increases from 0 to 1, while the price sensitivity to $\pi_t$, measured by $p_\pi$, decreases from 3.8095 to 0. Indeed, the price does not depend on $\pi_t$ when no one in the market can observe it. The price sensitivity to $\theta_t$'s, the parameter $p_\theta$, is negative and increasing in magnitude as the number of technical traders increases. This is due to the fact that technical investors infer the combination of $\Lambda \equiv p_\pi \pi + p_\theta \theta$ from the price. If they assign smaller portion of $\Lambda$ to $\pi$, they must infer from the price that a larger portion of the $\Lambda$ is contributed by $\theta$, hence larger (in magnitude) the $p_\theta$. The constant parameter $p$ is not monotone in relation to $w$, it increases from 6.8209 at $w = 0$ to 21.6172 at $w = 0.5$, and then decreases to 4.8052 when $w = 1$.

The most interesting part is the price sensitivity to $MV_t$, the parameter of $p_{mv}$, which is relatively small due to the low regression parameter $\beta_3$ in the regression of (18) that the technical traders use to infer $\pi_t$, making the technical traders more conservative in asset allocation based on MA. The small sensitivity to MA in the regression, and hence small $p_{mv}$, can be altered by assuming non-zero correlations among state variables. Alternatively we can assume that the technical traders use the MA rule in a different way. However, it is important to note that these various assumptions will not alter one of the main insights of the model that MA signal is priced. Moreover, if we increase the MA window and set $\alpha = 0.1$, the price sensitivity to $MV_t$ is much increased, as shown by Panel B of Table I.

## B Trading Strategy and Equilibrium Characterization

To understand the price behavior in equilibrium, in particular the term involving $MV_t$, we need to understand the trading strategy of each group of investors. In our setting, we define trend-following (contrarian) strategy as a positive (negative) feedback strategy in the sense of De Long et al. (1990). Hence, a strategy positively (negatively) correlated to the $MV_t$ is a trend-following (contrarian) strategy. It is important to keep in mind that the equilibrium price solution is a stationary one, hence the aggregate behavior of the market has to be contrarian, otherwise the price will go extreme (as evidenced by the market crash of 1987 where the trend-following trades increased dramatically due to portfolio insurance). When market is dominated by informed investors, the market price can sustain certain portion of trend-following strategy by technical traders as long as the majority, the informed investors, are contrarian; when market is dominated by technical traders, they have to be contrarian otherwise the price blows up in the long run. These features are demonstrated in the results in Table II.

It is important to notice the sign of $p_{mv}$ in Table I. When $\alpha = 1$, $p_{mv}$ is positive for small $w$ and becomes negative when $w$ increases to 0.3. The positive $p_{mv}$ implies a trend-following price behavior, while negative $p_{mv}$ implies a contrarian. The portfolio demand by informed and technical investors are presented in Panel A of Table II. In the table, we express the portfolio demand in terms of $MV_t$ and $\Lambda_t$, which is defined as

$$\Lambda_t = P_t - (p - p_D D_t - p_{mv} MA_t),$$

hence the portfolio demand by informed investors of Equation (17) and technical traders of Equation (37) can be expressed as

$$\begin{aligned} \eta^i &= g_0^i + g_D^i D_t + g_\pi^i \pi_t + g_\theta^i \theta_t + g_{mv}^i MV_t, \\ \eta^u &= g_0^u + g_D^u D_t + g_\lambda^u \Lambda_t + g_{mv}^u MV_t, \end{aligned} \tag{43}$$

where $g^i$'s and $g^u$'s are the demand loadings on the state variables.

There are a few interesting observations from Table II. First, in equilibrium, informed traders have positive (negative) demand loading on $\pi_t$ or $D_t$, since high $\pi_t$ or $D_t$ corresponds to good (bad) investment opportunity for them. On the other hand, technical traders are uninformed of $\pi_t$ and can only infer $\pi_t$ from $D_t$, $P_t$ and $MV_t$. To the extent that they perceive high $D_t$ associated with high $\pi_t$ as demonstrated by a positive regression coefficient $\beta_1$ in Equation (18), their asset demand demonstrates positive loading on $D_t$.

Second, $g_\theta^i$ increases as $w$ increases, which means that in equilibrium, when noise traders sell (buy), the informed traders buy (sell) more as the number of technical traders increases. This is due to the fact that the technical traders tend to trend follow the noise traders because they cannot differentiate liquidity trades from fundamentals. Hence the informed traders tend to accommodate trades from both noise traders and technical traders. As a result, the price is more sensitive to the liquidity trades $\theta_t$ and more rent is extracted from the noise traders in the presence of the technical traders.

Third, as discussed previously, for informed traders, $MV_t$ is the investment opportunity with respect to the risk factor $A_t$ in price equation (7), hence high (low) $MV_t$ indicates good (bad) state when price is positively (negatively) loaded with $A_t$, i.e., $p_{mv} < 0(> 0)$ given equation (42), while for technical traders the opposite has to be true to clear the market. To the extent that the technical traders infer $\pi_t$ from $A_t$, for positive (negative) $\beta_3$, high $A_t$ represents good (bad) state for them. Hence it must be the case that positive (negative) $\beta_3$ necessarily implies negative (positive) $p_{mv}$, which is indeed true by examining $p_{mv}$ in Panel A of Table I and the $\beta_3$ in Panel B of Table II, which shows numerically the regression coefficients of Equation (18) and (19) used by technical traders. Indeed, the tables show that when $w < (\geq)\ 0.3$, $\beta_3$ is negative (positive), while $p_{mv}$ is positive (negative).

Fourth, the table also shows that when $w < (\geq)\ 0.3$, $g^u_{mv} > (<)\ 0$ and $g^i_{mv} \leq (>)\ 0$, which says for technical traders, when the ratio of technical traders are small, the usual trend following MA strategy is profitable; when more investors are using the trend following MA strategy, it becomes unprofitable and the opposite is true. This validates our previous statement that trend following strategy is intrinsically destabilizing, and hence only a small portion of trend following traders can be sustained. In other words, in equilibrium not all traders can be trend following.

It is interesting to compare the mechanism of the survival of the technical traders in our model to related literature. In one aspect, the survival mechanism of technical traders in our model is similar to the noise traders in De Long, Shleifer, Summers, and Waldmann (1991). They point out that "since noise traders who are on average bullish bear more risk than do investors holding rational expectations, as long as the market rewards risk taking such noise traders can earn a higher expected return even though they buy high and sell low on average. The relevant risk need not even be fundamental: it could simply be the risk that noise traders' asset demands will become even more extreme tomorrow than they are today and bring losses to any investor betting against them." In our model, as can be verified in Table II, informed investors reduce their asset holding as the ratio of technical traders increases. This is due to the fact that the risk increases in the presence of technical traders, and they rationally reduce the demand for the risky asset as a consequence. Since technical traders perform the function of risk sharing in the presence of noise traders, they survive in the long run.

On the other hand, in a setting with rational and irrational traders, Kogan, Ross, Wang, and Westerfield (2006) examine the price impact and survival of irrational traders. In contrast to their model where the irrational traders lose money to rational traders, as demonstrated above, both informed and technical traders make money from noise traders who pay for liquidity demand in our model in a mechanism similar to Cespa and Vives (2012). However, unlike Cespa and Vives (2012), where the informed traders could either accommodate or follow the noise traders depending on the persistence of the noise trades, the informed traders in our model always accommodate the noisy traders while the technical traders always follow them. This is because in their model the informed traders who have heterogeneous information may speculate against each other, which is prevented in our model since they all have the same information set. As a result, the price becomes even lower (higher) when noise traders sell (buy) than the case of no technical traders, and hence in aggregate they can extract more money from the noise traders than otherwise.

In summary, the equilibrium is reached when both informed and technical traders are compensated for bearing fundamental risks and for providing liquidity. This is possible due to the liquidity demand by noise traders. As a result, the MA is a priced risk factor, which we empirically demonstrate via a trend factor portfolio based on the MA factor in the next section.

# V Trend Factor

## A Data and Methodology

We use the *daily* stock prices of the Center for Research in Security Prices (CRSP) from January 1926 to December 2013 to calculate the trend signals only. Based on them, we form our portfolios and factors, and rebalance them at the usual *monthly* frequency. We include all domestic common stocks listed on the NYSE, AMEX, and Nasdaq stock markets, and exclude closed-end funds, real estate investment trusts (REITs), unit trusts, American depository receipts (ADRs), and foreign stocks (or stocks that do not have a CRSP share code of 10 or 11). In addition, we exclude stocks with price below $5 (price filter) and stocks that are in the smallest decile sorted with NYSE breakpoints (size filter). Jegadeesh and Titman (1993) use the same price and size filters when constructing the momentum strategy. A relaxation of either or both of the filters and other alternative procedures will be examined in Section VI.

To construct the trend factor, we first calculate the MA prices on the last trading day of each month. The MA on the last trading day of month $t$ of lag $L$ is defined as

$$A_{jt,L} = \frac{P^t_{j,d-L+1} + P^t_{j,d-L+2} + \cdots + P^t_{j,d-1} + P^t_{j,d}}{L}, \tag{44}$$

where $P^t_{jd}$ is the closing price for stock $j$ on the last trading day $d$ of month $t$, and $L$ is the lag length. Then, we normalize the moving average prices by the closing price on the last trading day of the month,

$$\tilde{A}_{jt,L} = \frac{A_{jt,L}}{P^t_{jd}}. \tag{45}$$

There are three reasons for this normalization. First, according to our theory (see Equation (8)), it is this normalized averages that predict the future stock returns. Second, econometrically, the normalization makes the MA signals stationary.[8] Third, the normalization can also mitigate the undue impact of the high priced stocks.

It is worth emphasizing that there are various economic forces that contribute to the price trends in the stock market that make the MA signals useful. Besides the theories reviewed in the introduction and our new model, Barberis, Shleifer, and Vishny (1998) argue that prices can trend slowly when investors underweight new information in making decisions. In addition, prices may rise steadily in reaction to future good news, and the price reaction can have not only self-fulfilling effects, but also the positive feedback effects (discusses in Section II). Moreover, any theory that explains momentum can be potentially useful for explaining the price trends. On the other hand, putting theory aside, simple economic intuition also suggests that markets or asset prices can trend from time to time perhaps due to business cycles or fads in consumer products and services. Often a new investment idea may take time to fruit. For example, Warren Buffet may take time to build a large position due to practical liquidity problem, and take yet another long period of time to realize the gain. In short, the MA signals are economically valid predictors of future stock prices, which are not only motivated explicitly by our model, but also supported generally by models on price trends and by simple economic arguments.

To predict the monthly expected stock returns cross-sectionally, we use a two-step procedure similar to Haugen and Baker (1996). In the first-step, we run in each month $t$ a

[8] Keim and Stambaugh (1986) use a similar strategy to make the S&P 500 index stationary.

cross-section regression of stock returns on the prior month trend (normalized MA) signals to obtain a time-series of the coefficients on the signals.[9]

$$r_{j,t} = \beta_{0,t} + \sum_i \beta_{i,t}\tilde{A}_{jt-1,L_i} + \epsilon_{j,t}, \quad j = 1, \cdots, n \tag{46}$$

where

$$\begin{aligned} r_{j,t} &= \text{rate of return on stock } j \text{ in month } t, \\ \tilde{A}_{jt-1,L_i} &= \text{trend signal at the end of month } t-1 \text{ on stock } j \text{ with lag } L_i, \\ \beta_{i,t} &= \text{coefficient of the trend signal with lag } L_i \text{ in month } t, \\ \beta_{0,t} &= \text{intercept in month } t. \end{aligned}$$

It should be mentioned that only information in month $t$ or prior is used above.

Then, in the second-step, we estimate the (out-of-sample) expected return for month $t+1$ from

$$E_t[r_{j,t+1}] = \sum_i E_t[\beta_{i,t+1}]\tilde{A}_{jt,L_i}, \tag{47}$$

where $E_t[r_{j,t+1}]$ is our estimated expected return on stock $j$ for month $t+1$, and $E_t[\beta_{i,t+1}]$ is the estimated expected coefficient of the trend signal with lag $L_i$, and is give by

$$E_t[\beta_{i,t+1}] = \frac{1}{12}\sum_{m=1}^{12} \beta_{i,t+1-m}, \tag{48}$$

which is the average of the estimated loadings on the trend signals over the past 12 months. Note that we do not include an intercept above because it is the same for all stocks in the same cross-section regression, and thus it plays no role in ranking the stocks below. Again, it is worth noting that only information in month $t$ or prior is used to estimate the expected returns at month $t+1$.

Now, following the standard procedure on cross-section studies, such as Jegadeesh and Titman (1993), Easley, Hvidkjaer, and O'Hara (2002), Ang, Hodrick, Xing, and Zhang (2006, 2009), and Han and Lesmond (2011, 2014), we sort stocks into portfolios by the expected returns forecasted from the trend signals. Specifically, in each month stocks are sorted into five quintiles according to their forecasted expected returns, and then we form an equal-weighted portfolio of the stocks in each of the quintiles. The sorting procedure thus produces

[9] Jegadeesh (1990) also uses a similar cross-sectional regressions to predict individual stock returns, but he uses past returns instead of trend indicators.

five quintile portfolios that are re-balanced every month. The High-Low spread portfolio is our trend factor, constructed as a zero-cost arbitrage portfolio that takes a long position in the highest ranked quintile portfolio (High) and takes a short position in the lowest ranked quintile portfolio (Low). Note that the trend factor is a portfolio formed out-of-sample, similar to many other factors used in the finance literature.

The popular momentum factor (Jegadeesh and Titman, 1993) can be interpreted as a special case of the above procedure. If there is only one trend signal, past year price, and if the beta is one, the trend factor coincides with the momentum factor. Of course, we use multiple signals to capture trends in various time horizons, short-, intermediate-, and long-term. Following, for example, Brock et al. (1992), we consider all the common trend indicators, the moving averages of lag lengths 3-, 5-, 10-, 20-, 50-, 100-, and 200-days. To include information in the longer term, we also add 400-, 600-, 800- and 1000-days. Intuitively, they indicate the daily, weekly, monthly, quarterly, 1-year, 2-year, 3-year, and 4-year price trends of the underlying asset. Note that we simply use all of these popular indicators without any alterations or removals. We do not use any complicated trend signals either. In other words, we have not attempted to obtain the greatest possible abnormal returns by optimizing the estimation procedure. Our reasoning is that the naive procedure might be more powerful to show the existence of genuine trends in the market than complicated procedures because the latter, though they could perform much better, are difficult to overcome the concern of data-mining. Nevertheless, we do examine, for robustness reasons, some alternative specifications that include various number of trend signals and shorter lag-lengths up to only to 200 days. The (unreported) results are largely unchanged or better, supporting the main conclusion of the paper that the trend factor matters, which is supported by further robustness analysis in Section VI.

## B Summary Statistics

In this subsection, we provide first the summary statistics of the trend factor and compare them with those of other common factors. Then we conduct the spanning tests. Finally, we examine the associated trend quintile portfolios and their general characteristics.

In order to compute the trend signals and estimate their expected coefficients, we have to skip the first 1000 days and subsequent 12 months. So the effective sample period for our study is from June, 1930 a total of 1002 observations, during which the trend factor is well defined.

Table III reports the summary statistics of the trend factor, short-term reversal factor (SREV), momentum factor (UMD), long-term reversal factor (LREV), as well as the Fama-French three factors (Market, SMB, and HML).[10] The average monthly return of the trend factor from June 1930 to December 2013 is 1.63% per month, or 19.60% per annum, more than doubling the average return of any of the other factors including SREV, whose average return is the highest among the other factors but is only 0.80% per month. The standard deviation of the trend factor is about 3.49%, which is lower than those of many of the other factors. As a result, the Sharpe ratio of the trend factor is much higher than those of the other factors. For example, the trend factor has a Sharpe ratio of 0.47, whereas the next highest Sharpe ratio is only 0.23 generated by SREV. Daniel and Moskowitz (2013) and Barroso and Santa-Clara (2012) show that returns generated from the momentum strategies are negatively skewed with large kurtosis, which implies a very fat left tail. Consistent with these results, Table III shows that the momentum factor has a large negative skewness ($-3.10$) and large kurtosis (27.8). In contrast, the trend factor has a large positive skewness (1.47) and large kurtosis (12.7), indicating a fat right tail, great chances for large positive returns.

It is of interest to see how the factors perform in bad times. Panel A of Table IV shows that the average return and volatility of the trend factor are both higher in the recession periods than in the whole sample period. The average return rises from 1.63% to 2.30%, and the volatility rises from 3.49 to 4.99%. However, the Sharpe ratio is virtually the same, from 0.47 to 0.46. In contrast, the momentum factor and Fama-French factors experience much lower returns and higher volatilities, and hence much lower Sharpe ratios. Interestingly, though, all of the other factors still have positive average returns, while the market suffers an average loss of $-0.68$% per month in recessions. Another interesting fact is that the momentum factor experiences the greatest increase, more than 69%, in volatility (from 4.82% to 8.14%) in recessions. Finally, consistent with Nagel's (2011) finding that short-term reversal strategies do well in recessions due to "evaporating liquidity", both the short-term reversal and trend factors perform better than over the entire sample period.

[10] Data on the trend factor will be posted, and the other factors are available from Ken French's online data library.

However, the performance of the trend factor is robust to recessions (see Table VI) because it is based on the MA signals of not only the short-term, but also the intermediate and long-term.

Panel B of Table IV reports the summary statistics for the most recent financial crisis period. The average return of the trend factor is about 0.79% per month, and the Sharp ratio is about 0.16. In contrast, all the other factors except SMB (0.63% per month) and LREV (0.03% per month) experience large losses: the market yields −2.03% per month, SREV yields −0.80% per month, and UMD yields −1.33% per month. In addition, the volatility of the UMD factor increases to 10.6%, an increase of more than 120% compared to the whole sample period. Interestingly, the skewness and kurtosis are generally smaller during the financial crisis period. However, the trend factor earns the lower average return during the financial crisis then the overall period.

Daniel, Jagannathan, and Kim (2012) show that the momentum strategy suffers loss exceeding 20% per month in 13 months out of the 978 months from 1929 to 2010. Barroso and Santa-Clara (2012) show that the momentum strategy delivers a −91.59% return in just two months in 1932 and a −73.42% over three months in 2009. This evidence motivates us to examine the worst case scenarios for the trend factor. Table V compares the maximum drawdown, Calmar ratio, and frequency of big losses of the trend factor with those of the other factors (Panel A). The maximum drawdown is defined as the largest percentage drop in price from a peak to a bottom, which measures the maximum loss of an investor who invests in the asset at the worst time. From June, 1930 to December, 2013, the maximum drawdown is 20.7% for the trend factor, 33.2% for SREV, 77.3% for UMD, and 46.3% for LREV, and 76.6% for the market. The Calmar ratio, widely used in the investment industry, is defined as the annualized rate of return divided by the maximum drawdown, which measures return versus downside risk. The higher the ratio, the better the risk-return tradeoff. From June, 1930 to December, 2013, the trend factor has a Calmar ratio of 94.7%, whereas the other four factors have much lower Calmar ratios. For example, the momentum factor yields a Calmar ratio of only 10.0%, and the long-term reversal factor delivers a Calmar ratio of only 9.01%.

Out of the 1002 months, the trend factor suffers a loss in 248 months, i.e., about 24.75% chance of negative returns from June 1930 to December 2013. In contrast, the other four factors all suffer greater number of losses. The least one among them is the momentum factor, which suffers 369 monthly losses or has 36.82% chance of suffering negative returns. For the extreme losses, out of the 1002 months, only in four months did the trend factor experience a negative return exceeding −10% and only in one month exceeding −20%. In contrast, the momentum factor experiences a negative return exceeding −10% in 17 months and exceeding −20% in six months. The worst is the market, which has 30 monthly losses more than −10% and has five months more than −20%.

Panel B of Table V reports the correlation matrix of the trend factor with the four factors. The trend factor is correlated with the short-term reversal factor (33%) and long-term reversal factor (13%), but the correlation is virtually zero with the momentum factor. It also has about 20% correlation with the market portfolio. It is worth noting that the momentum factor is the only factor that is negatively correlated with the market (−35%).

Why does the trend factor have a virtually zero correlation with the momentum factor? Indeed, this seems at first glance somewhat puzzling since both factors are trend capturing. However, this becomes easy to understand once we separate the long and short sides of both factors. Table VI reports the summary statistics for their long and short portfolios.[11] Due to capturing overall the same trend, the long (short) portfolios of both factors are indeed positively correlated as expected, and the correlation is in fact as high as 89% (84%) (Panel A). However, the trend factor does a much better job in capturing the trend, so it has a much greater average return, 2.32% versus 1.84% of the momentum factor for the long leg, and has a much smaller average return, 0.69% versus 1.04% for the short leg.[12] The differences are statistically significant at the 1% level as shown in the second to the last column of the table. This means that both long and short portfolios of the trend factor outperform those of the momentum factor, and so the trend factor, as the spread portfolio, must outperform the momentum factor.[13] Moreover, we find that the trend factor earns a much higher average return during recessions while the momentum factor earns a small positive average return only because the short leg is a bit more negative than the long leg (Panel B). This is why

[11] The long and short portfolios of the momentum used are the $10^{th}$ and first equal-weighted decile portfolios taken from Ken French's online data library. Results are similar if we use the size and momentum double sorted portfolios that constitute the UMD factor.

[12] Note that we are somewhat biased against the trend factor as the quintile portfolios in contrast to the decile portfolios of the momentum are used. If trend deciles are used, the long and short portfolios will have average returns of 2.40% and 0.24% per month, respectively, thus yielding an average return of 2.16% per month for the decile trend factor.

[13] The spread portfolio between the $10^{th}$ momentum decile and the first momentum decile has an average return of 0.8% per month, close to 0.7% of the UMD factor.

returns on the two factors are not correlated over the entire sample period. In expansion periods as shown in Panel C, the trend factor still earns a higher average return than does the momentum factor because both long and short legs of the trend factor outperform those of the momentum factor.

## C Mean-Variance Spanning Tests

Since our trend factor uses information on the short-term, intermediate-term and long-term stock trends, and since the three trends are traditionally captured by the short-term reversal, momentum, and long-term reversal factors, it is a logical question whether a portfolio of the three factors can mimic the performance of the trend factor. For example, can the trend factor outperform an equal-weighted portfolio of the three factors? In fact, the trend factor outperforms *any* portfolio of the three factors in terms of the Sharpe ratio.

To see why, it is sufficient to show that the trend factor lies outside the mean-variance frontier of the three factors. Huberman and Kandel (1987) is the first to provide a mean-variance spanning test on the hypothesis that whether $N$ assets can be spanned or replicated in the mean-variance space by a set of $K$ benchmark assets. De Santis (1993), Bekaert and Urias (1996), De Roon, Nijman, and Werker (2001), Korkie and Turtle (2002), Kan and Zhou (2012) provide additional tests of the same hypothesis. Statistically, we run regression of the trend factor on the other three factors,

$$f_{0t} = \alpha + \beta_1 f_{1t} + \beta_2 f_{2t} + \beta_3 f_{3t} + \epsilon_t, , \tag{49}$$

where $f_{0t}$, $f_{1t}$, $f_{2t}$ and $f_{3t}$, are the returns on the trend factor, the short-term reversal, momentum, and long-term reversal factors. The spanning hypothesis is equivalent to the following parametric restrictions on the model,

$$H_0: \quad \alpha = 0, \quad \beta_1 + \beta_3 + \beta_3 = 1. \tag{50}$$

Following Kan and Zhou (2012), we carry out six spanning tests: Wald test under conditional homoscedasticity, Wald test under independent and identically distributed (IID) elliptical distribution, Wald test under conditional heteroscedasticity, Bekerart-Urias spanning test with errors-in-variables (EIV) adjustment, Bekerart-Urias spanning test without the EIV adjustment and DeSantis spanning test. All six tests have asymptotic Chi-Squared distribution with $2N$ ($N = 1$) degrees of freedom.

Table VII reports the results for the whole sample, recession and financial crisis periods. The hypothesis is rejected strongly that the trend factor is inside the mean-variance frontier of the short-term reversal, momentum, and long-term reversal factors for the whole sample and recession periods. For the financial crisis period, due to the much smaller sample size, the power of the tests should be weakened substantially. Nevertheless, even in this period, the hypothesis is still rejected at the 5% level. Overall, the trend factor is clearly a unique factor that captures the cross-section of stock trends and performs far better than the well known short-term reversal, momentum, and long-term reversal factors.

## D The Trend Quintile Portfolios

Table VIII reports the average returns and characteristics of the equal-weighted quintile portfolios sorted by the expected returns forecasted using the trend signals (trend quintile portfolios). The average returns increase monotonically from the quintile with the lowest forecasted expected returns (Low) to the quintile with the highest forecasted expected returns (High). More specifically, stocks forecasted to have the highest expected returns (strongest trend forecasts) yield the highest returns on average in the subsequent month, about 2.32% per month, whereas stocks forecasted to have the lowest expected returns (weakest trend forecasts) yield the lowest returns on average in the subsequent month, only about 0.69% per month.

Also worth noting are the large gaps in average returns between the lowest and the second quintiles and the highest and the fourth quintiles - the average return increases by 58% (0.40% per month) and 44% (0.71% per month), respectively.

The market size displays a hump shape across the quintiles - both quintile Low and High have smaller market size than the other quintiles, while the book-to-market (B/M) ratio stays roughly constant across the quintiles. Not surprisingly, the prior month returns ($R_{-1}$) decrease monotonically across the quintiles, whereas the past six-month cumulative returns ($R_{-6,-2}$) increase monotonically across the quintiles. Idiosyncratic volatility (Idio Vol) displays a U-shaped pattern across the quintiles - the two extreme quintiles have much higher idiosyncratic volatility. We also report the percentage of zero returns (%Zero) and share turnover rate, both of which measure the liquidity of stocks (Lesmond, Ogden, and Trzcinka, 1999). While the percentage of zero returns stays roughly constant across quintiles, the turnover rate displays a U shape - the turnover rate is higher for both extreme quintiles. The last two columns in Table VIII report price ratios. Both the earnings-to-price ratio (E/P) and the sales-to-price ratio (S/P) display a hump-shaped pattern across the quintiles.

## E Risk Adjusted Returns

Can common risk factors explain the return on the trend factor? Table IX reports Jensen's alpha and risk loadings for the trend quintile portfolios, the trend and momentum factors, with respect to the CAPM and Fama-French three-factor model, respectively. The quintile alphas increase monotonically from the lowest quintile to the highest quintile, from −0.32% to 1.23% with respect to the CAPM, and from −0.46% to 1.09% with respect to the Fama-French three-factor model, respectively. As a result, the trend factor, which is the High-Low spread portfolio, has a CAPM alpha of 1.56% per month, and a Fama-French alpha of 1.55% per month, only slightly lower than the unadjusted abnormal return (1.63% per month in Table III). While the market beta and SMB beta are U-shaped, the HML beta is hump-shaped across the quintiles. Hance the trend factor has a small loading on the market and insignificant SMB and HML betas in the Fama-French three-factor model.

Schwert (2003) finds that, out of all the existing major anomalies, the momentum is the only one that is alive after its publication. Impressively, it also survives the high hurdle proposed by Harvey, Liu, and Zhu (2013) to detect false discoveries. In comparison with the momentum, the alpha of the trend factor is almost twice as large, and has a t-statistic of 13.7 vs 7.24. Hence, the trend factor appears more reliable statistically and more anomalous economically than the momentum factor.[14]

# VI Robustness

In this section, we show that the superior performance of the trend forecasts is robust to both alternative formations and to major financial markets. We present more robust tests in the online appendix, including controlling for various firm characteristics such as size, book-

[14]Standard asset pricing models, such as the consumption-habitat model of Campbell and Cochrane (1999) and the long-run risks model of Bansal and Yaron (2004), are unlikely capable of explaining the trend factor, similar to the momentum factor case. A simple intuitive reason is that the information in the MAs are often not in common macroeconomic variables (see, e.g., Neely et al. (2014).)

to-market ratio, past returns, percentage of zero returns, etc., using the alternative Fama-MacBeth regression methodology, and interacting with the various information uncertainty proxies such as idiosyncratic volatility, analyst coverage, and firm age.

## A Alternative Formations

Recall that we have excluded stocks with price below \$5 (price filter) and stocks that are in the smallest decile sorted with NYSE breakpoints (size filter). An interesting question is whether the removal of one or both filters can substantially worsen the results. Table X provides the results. When no filters are applied, the trend factor will have a much greater average return of 2.92% per month instead of 1.63% per month. Moreover, the volatility is slightly higher (4.54%), and the Sharpe ratio is considerably higher (0.64) compared to the early value of 0.47. The skewness and kurtosis are higher too.

Now if we impose only the size filter, the average return is 1.97%, the volatility is 4.12%, and the Sharpe ratio is 0.48, making the trend factor better. Similarly, if we impose only the price filter, the average return is 1.80%, the volatility is 3.38%, with a higher Sharpe ratio of 0.53 than before. Therefore, the filters we impose on the trend factor are not to make it better, just to make it more implementable. The results also seem to suggest that small stocks and low priced stocks are more trending. This may be intuitively clear as large stocks have more analysts following and more investors, and hence more information transparency and faster price movements to reflect all available information.

Fama and French (1993) use a double sorting procedure to construct their well-known factors. It seems of interest to investigate how our trend factor will perform if it is constructed by their approach. Following Fama and French (1993), we first sort the stocks into two groups by NYSE size breakpoints, and then independently sort the stocks into three groups by the forecasted expected returns from the trend indicators. The breakpoints are the 30th and 70th NYSE percentiles. The trend factor is then the average of the two portfolios with the high forecasted expected returns minus the average of the two portfolios with the low forecasted expected returns,

$$Trend = \frac{1}{2}(Small\ High + Big\ High) - \frac{1}{2}(Small\ Low + Big\ Low). \tag{51}$$

The average return of the formed trend factor is higher (1.91% per month), the standard deviation is lower (3.38% per month), and thus the Sharpe ratio is higher than those reported in Table III (0.60 versus 0.47).[15] The skewness and kurtosis are higher as well.

## B International Evidence

In this subsection, we further provide international evidence on the profitability of the trend factor. To this end, we extract daily prices and monthly returns from Datastream for the other G7 (G6) countries - France, United Kingdom (UK), Germany, Italy, Canada, and Japan. We then form the trend factor for each country using the same procedure used for the US.

Similar to Table III, Table XI reports the summary statistics of the trend factor, the short-term reversal, momentum, and long-term reversal factors,[16] as well as the CAPM alpha relative to the global market portfolio, for the available sample period from January, 1990 to December, 2013. The trend factor yields significant and positive average returns in all G6 countries, ranging from 0.73% per month in UK to 2.07% in Italy. In contrast, the momentum factor is insignificant in both Italy and Japan. The other two reversal factors are mostly insignificant: the short-term reversal factor is only significant in Japan and marginally significant in France, and the long-term reversal factor is only significant in UK and Japan. In four of the G6 countries where the momentum factor is significant, the trend factor underperforms the momentum factor, but the underperformance is statistically insignificant, whereas in the other two countries, the trend factor outperforms the momentum, and in particular, the outperformance in Italy is statistically significant. The underperformance is likely due to the weak performance of the short-term and long-term reversals because the trend factor utilizes information of both the short-term and long-term MA signals in addition to the intermediate-term MA signals.[17] In terms of higher moments, the results are similar to the US case. For example, the trend factor almost always has a positive skewness (except for Japan (-0.03) and UK (-0.00)), whereas the momentum factor almost always

[15]Table X reports only the equal-weighted results, but the value-weighted results are similar and are just slightly weaker. For example, using the Fama and French approach, the value-weighted trend factor delivers an average return of 1.29% per month (as highly significant as the equal-weighted), a standard deviation of 2.90% per month, and thus a Sharpe ratio of 0.44, slightly lower than 0.47 reported in Table III.

[16]The momentum factor is based on past 12-month cumulative returns skipping the last month; the long-term reversal is based on cumulative returns from month $t-25$ to $t-60$.

[17]If optimal statistical model selection rules are used to form a new trend factor, the new one is likely to do the best. However, for simplicity and robustness, we keep the naively formed trend factor in this paper.

has a negative skewness (except for Germany, which has a positive but very small skewness, 0.06). The trend factor is also much less volatile than the momentum factor in every G6 market. Finally, the patterns of the CAPM alphas are the same as those for the average returns.

Table XII reports the same statistics during the recent financial crisis period (12/2007 - 6/2009). Given the short time period and high volatility, all factors are insignificant except the trend factor in France and the long-term reversal factor in Japan. The most notable fact is that the average returns for the trend factor remains positive for all G6 countries. The momentum factor, on the other hand, yields much worse performance. For example, in UK, Canada and Japan, the momentum factor suffers large losses of $-3.61\%$, $-4.91\%$ and $-3.89\%$ per month, respectively, and thus the outperformance by the trend factor are statistically significant. In short, it appears that the trend factor is viable not only in the US, but also around the world.

# VII Cross-Section Pricing

In this section, we examine how well the trend factor can explain the cross-section portfolios sorted by the short-term reversal or by the short-term reversal, momentum, and long-term reversal simultaneously, as compared with the popular momentum factor.[18]

Consider first the short-term reversal decile portfolios. We investigate how they are explained by the CAPM or by the CAPM with the trend factor or by the CAPM with the momentum factor, respectively. Panel A of Table XIII reports the CAPM results. All but three short-term reversal portfolios have highly significant alphas. The first portfolio (*STRev1*), which contains the lowest last month returns, yields the highest abnormal returns (1.61% per month), whereas the $10^{th}$ portfolio (*STRev10*), which contains the highest last-month returns, yields the lowest abnormal returns ($-0.99\%$ per month). To assess further the cross-section overall pricing errors, we also provide a weighted summary of the alphas,

$$\Delta = \alpha' \Sigma^{-1} \alpha, \tag{52}$$

where $\Sigma$ is the variance-covariance matrix of the residuals across the ten decile portfolios.

[18]While the trend factor has smaller aggregate pricing error than the momentum factor for portfolios sorted in many ways, we focus here on these two for brevity.

Shanken (1992) seems the first to introduce this measure (different from his paper by a scalar) which is related to the optimal portfolio that exploits the pricing errors. The larger the $\Delta$, the greater the difference between investing in the factor portfolio(s) and all the decile portfolios used on the left-hand side of the regressions. The second to the last column of Panel A reports a value of 0.28 per month. Economically, to a mean-variance investor with a risk aversion of five, this translates to an annual utility gain of 33.6% ($= 0.28 \times 12/(2 \times 5)$) over investing in the factor(s) alone. The large value is due partly to the large alphas and partly to the assumption that the sample mean and covariance matrix are the true parameters. The last column reports the GRS test of Gibbons, Ross, and Shanken (1989), which rejects the hypothesis strongly that all the alphas are zero or that the CAPM can explain the short-term reversal decile portfolios.

Panel B of Table XIII reports the results when the trend factor is added to the CAPM regressions. The number of significant alphas at the 5% level has been reduced from seven to four. The loadings on the trend factors across all the ten deciles are significant (except for the $6^{th}$ decile), and decrease more or less monotonically from the first decile to the $10^{th}$ decile. That is, the decile portfolio that has the lowest last-month returns has the highest positive exposure to the trend factor, whereas the decile portfolio that has the highest last-month returns has the highest negative exposure to the trend factor. In contrast, all but three decile portfolios have highly significant pricing errors when the momentum factor is added to the CAPM regressions instead of the trend factor (Panel C). Moreover, the magnitude of the pricing error is 0.31, which is about 35% greater than 0.23 of the CAPM with trend factor. However, the CAPM with the addition of either the trend or the momentum factor is still rejected strongly by the GRS test.[19] Nevertheless, the trend factor performs better in explaining the returns than the momentum factor.

Consider now the portfolios that are formed based on sorting stocks independently by the short-term reversal (last month returns, $R_{-1}$), the momentum (past 12 month cumulative returns skipping the last month, $R_{-12,-2}$), and the long-term reversal (past 60 month cumulative returns from month $t-25$ and $t-60$, $R_{-60,-25}$). We form $3 \times 3 \times 3$ equal-weighted portfolios and then regress the monthly portfolio excess returns on the market excess returns (CAPM), the market excess returns and the trend factor returns (CAPM plus Trend Factor),

[19] If the CAPM is replaced by the Fama-French three-factor model, the results are virtually unchanged. We omit them here for brevity.

and the market excess returns and the momentum factor returns (CAPM plus Momentum Factor), respectively. Panel A of Table XIV reports the CAPM alphas, which range from $-0.61\%$ for portfolio 21, corresponding to the highest last month return, lowest past 12 month return, and highest past 60 month return, and 1.13% for portfolio 7, corresponding to the lowest last month return, highest past 12 month return, and lowest past 60 month return. Only four out of 27 alphas are insignificant. The cross-section pricing error ($\Delta$) is 0.35 and GRS statistic is 12.5 and highly significant.

In contrast, adding the trend factor into the CAPM, as shown in Panel B, helps to reduce all the alphas, which now range from $-0.28\%$ (portfolio 21) and 1.01% (portfolio 8), and seven out of 27 alphas are insignificant. The cross-section pricing error $\Delta$ is also reduced from 0.35 to 0.21, and the GRS statistic is about half, 6.57. On the other hand, Panel C shows that adding the momentum factor reduces some alphas but increases others – alphas now range from $-0.31\%$ (portfolio 21) to 1.31% (portfolio 1). As a result, the cross-section pricing error is only reduced slightly, from 0.35 to 0.33 and GRS statistic from 12.5 to 11.6.

Overall, in comparison with the popular momentum factor, the trend factor not only has three times greater the Sharpe ratio, but also performs better in explaining the cross-section returns. Hence, it appears that the trend factor offers an interesting complement to the popular momentum factor.

# VIII Conclusions

While technical trading is pervasive in practice, there is a lack of understanding of its role in asset pricing. This paper provides the first equilibrium model for the pricing role of technical traders who use primarily the moving averages of past prices. In our model in which noise traders have inelastic liquidity demand, the informed investors can extract more "rents" from providing liquidity to the noise traders than in an economy in absence of the technical traders. Hence, the technical traders in our model can survive and earn an equilibrium return for providing a risk-sharing function in the market. As a consequence, the moving average signal used by technical traders is a priced risk factor which depends on the population ratio of technical traders in the market. When the ratio is small, technical traders will behave as trend-followers, while when it is large, they can be contrarians.

Our model provides a theoretical explanation to both predictability of the moving average (MA) of past stock prices and the time series momentum (TSMOM). Because of technical trading, the model not only explains the predictability of the MA, but also identifies explicitly the functional form of the MA that predicts returns. Due to a common lag of information processing by technical traders, the model also explains Moskowitz et al. (2012)'s finding that the market prices tend to be positively correlated in the short-run and negatively correlated in the long-run, and that the past 12 month return predicts the future next month return across asset classes.

Based on our model, we form a trend factor that makes use of multiple MA trend indicators containing daily, weekly, monthly and yearly information. Stocks that have high forecasted expected returns from the cross-section regression yield higher future returns on average, and stocks that have low forecasted expected returns tend to yield lower future returns on average. The difference between the highest ranked and lowest ranked quintile portfolios is the trend factor, which has a return around 1.63% per month. The return is more than quadrupling those of the size and book-to-market factors, and more than twice that of the short-term reversal, momentum, and long-term reversal factors. The factor is robust to alternative formations and performs well in other G7 countries.[20]

Theoretically, our exploratory model has a number of simplifying assumptions, it will be of interest to extend the model by relaxing some of them. For example, it will be interesting to examine price as well as volume when the informed investors have differential information. In addition, it will be important to allow the portion of technical traders to change over time. Empirically, future research are called for to examine whether similar trend factors exist in other asset classes, such as foreign exchanges, commodities and bonds where the momentum is known to exist (see, e.g., Moskowitz et al., 2012; Asness, Moskowitz, and Pedersen, 2013).

[20]One internal report of a major investment bank finds that the trend factor works well in many other countries too.

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# Appendix

In this Appendix, we provide detailed proofs of the theoretical results.

## A Proof of Lemma 1

The state variables of the economy are given by

$$\begin{aligned} dD_t &= (\pi_t - \alpha_D D_t)dt + \sigma_D dB_{1t}, \\ d\pi_t &= \alpha_\pi(\bar{\pi} - \pi_t)dt + \sigma_\pi dB_{2t}, \\ d\theta_t &= -\alpha_\theta \theta_t dt + \sigma_\theta dB_{3t}. \end{aligned} \tag{A.1}$$

The variable vector that determines the informed investors' opportunity set is

$$\Psi = (1, D_t, \pi_t, \theta_t, A_t)^T, \tag{A.2}$$

which satisfies the following vector SDE,

$$d\Psi = e_\Psi \Psi dt + \sigma_\Psi dB_t, \tag{A.3}$$

where $B_t$ is a 5-dimensional Brownian Motion, $e_\Psi$ and $\sigma_\Psi \in R^{5\times5}$ are constant matrices,

$$e_\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_D & 1 & 0 & 0 \\ \alpha_\pi\bar{\pi} & 0 & -\alpha_\pi & 0 & 0 \\ 0 & 0 & 0 & -\alpha_\theta & 0 \\ p_0 & p_1 & p_2 & p_3 & p_4 - \alpha \end{pmatrix}, \tag{A.4}$$

and

$$\sigma_\Psi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_D & 0 & 0 & 0 \\ 0 & 0 & \sigma_\pi & 0 & 0 \\ 0 & 0 & 0 & \sigma_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A.5}$$

The investment opportunity is obtained as

$$dQ = (D_t - rP_t)dt + dP_t \equiv e_Q \Psi dt + \sigma_Q dB_t, \tag{A.6}$$

by differentiating Equation (7), where $e_Q$ and $\sigma_Q$ are

$$e_Q = (p_0(p_4 - r) + p_2\alpha_\pi\bar{\pi}, 1 + p_1(p_4 - r - \alpha_D), p_1 + p_2(p4 - r - \alpha_\pi), p_3(p_4 - r - \alpha_\theta), p_4(p_4 - r - \alpha)), \tag{A.7}$$

and

$$\sigma_Q = \begin{pmatrix} 0 & p_1\sigma_D & p_2\sigma_\pi & p_3\sigma_\theta & 0 \end{pmatrix}. \tag{A.8}$$

The result implies Lemma 1. QED.

## B Proof of Proposition 1

To prove Proposition 1, we conjecture a solution for the portfolio demand of the informed investors as a linear function of state variables as in Equation (17), and conjecture accordingly the value function of the following form,

$$J^i(W^i, D_t, \pi_t, \theta_t, A_t; t) = -e^{-\rho t - rW - \frac{1}{2}\Psi^{iT} V^i \Psi^i}. \tag{A.9}$$

Substituting this into the HJB equation (15), we obtain

$$\eta = f^i \Psi, \tag{A.10}$$

where

$$f^i = \frac{1}{r}(\sigma_Q \sigma_Q^T)^{-1}(e_Q - \sigma_Q \sigma_\Psi^T V^i) \tag{A.11}$$

with $V^i$ a symmetric positive satisfying

$$V^i \sigma_\Psi \sigma_\Psi^T V^{iT} - (\sigma_Q \sigma_Q^T)^{-1}(e_Q - \sigma_Q \sigma_\Psi^T V^i)^T (e_Q - \sigma_Q \sigma_\Psi^T V^i) + rV^i - (e_\Psi^T V^i + V^i e_\Psi) + 2k\delta_{11}^{(5)} = 0, \tag{A.12}$$

$k \equiv [(r-\rho) - r\ln r] - \frac{1}{2}Tr(\sigma_\Psi^T \sigma_\Psi V^i)$ and

$$[\delta_{(11)}^{(5)}]_{ij} = \begin{cases} 1, & i = j = 1 \\ 0, & \text{otherwise.} \end{cases} \tag{A.13}$$

This yields Proposition 1. QED.

## C Computation of $\beta_i$'s and $\gamma_i$'s

Based on Equation(18) the regression slope, $\beta \equiv (\beta_1, \beta_2, \beta_3)$, is given by

$$\beta = \text{Var}^{-1} \cdot \text{Cov}, \tag{A.14}$$

where $\text{Var} \in R^{3\times3}$ and $\mu \in R^{3\times1}$ be the variance and mean of vector $Y_t = (D_t, P_t, A_t)$, and $\text{Cov} \in R^{1\times3}$ the covariance between $\pi_t$ and $(D_t, P_t, A_t)$. Consider first how to compute Var and Cov. Given the price relation (7), we have $Y = FX$ with

$$F \equiv \begin{pmatrix} 0 & 0 & 1 & 0 \\ p_2 & p_3 & p_1 & p_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{A.15}$$

Then it follows that

$$\text{Var} = FCF^T, \quad \mu = Fm^T, \quad \text{Cov} = e_1 CF^T, \tag{A.16}$$

where $e_1 \equiv (1,0,0,0)$.

To compute the mean and covariance matrix of $X_t = (\pi_t, \theta_t, D_t, A_t)$, we obtain its dynamics through (A.1) and (7),

$$\begin{aligned} d\pi_t &= \alpha_\pi(\bar{\pi} - \pi_t)dt + \sigma_\pi dB_{2t}, \\ d\theta_t &= -\alpha_\theta \theta_t dt + \sigma_\theta dB_{3t}, \\ dD_t &= (\pi_t - \alpha_D D_t)dt + \sigma_D dB_{1t}, \\ dA_t &= [p_0 + p_1 D_t + p_2 \pi_t + p_3 \theta_t + (p_4 - \alpha)A_t]dt, \end{aligned}$$

which is an affine system. Let $\alpha_1 \equiv \alpha - p_4$. We now consider the following transform of $X_t$,

$$\Phi(u, x, t, T) = E_t[e^{u \cdot X_T}] = e^{A(t) + B(t) \cdot X_t}, \tag{A.17}$$

where $A(t)$ and $B(t)$ satisfy the ODE system (Duffie, Pan, and Singleton, 2000) below,

$$\frac{dB(t)}{dt} = -K_1^T B(t), \qquad B(T) = u, \tag{A.18}$$

$$\frac{dA(t)}{dt} = -K_0 \cdot B(t) - \frac{1}{2} B(t)^T H_0 B(t), \qquad A(T) = 0, \tag{A.19}$$

with

$$K_0 = \begin{pmatrix} \alpha_\pi \bar{\pi} \\ 0 \\ 0 \\ p_0 \end{pmatrix}, \; K_1 = \begin{pmatrix} -\alpha_\pi & 0 & 0 & 0 \\ 0 & -\alpha_\theta & 0 & 0 \\ 1 & 0 & -\alpha_D & 0 \\ p_2 & p_3 & p_1 & -\alpha_1 \end{pmatrix}, \; H_0 = \begin{pmatrix} \sigma_\pi^2 & 0 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 & 0 \\ 0 & 0 & \sigma_D^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A.20}$$

Note that the elements of covariance matrix of $X_t$ are the quadratic terms of $X_t$ in Taylor expansion of Equation (A.17).

To solve (A.18) and (A.19) analytically, we denote by $U$ and $\Lambda$ the eigenvectors and eigenvalues of $K_1^T$, i.e.,

$$U K_1^T = \Lambda U, \quad U^{-1} U = U U^{-1} = I, \quad \Lambda = diag(\lambda_1, ..., \lambda_4). \tag{A.21}$$

Then, due to the special form of $K_1^T$, the eigenvalues are all negative,

$$\lambda_1 = -\alpha_\pi, \quad \lambda_2 = -\alpha_\theta, \quad \lambda_3 = -\alpha_D, \quad \lambda_4 = -\alpha_1.$$

Therefore, we have

$$B_t = U^{-1} e^{\Lambda(T-t)} U u, \tag{A.22}$$

and

$$
\begin{aligned}
A_t &= \int_0^t K_0^T B_s ds + \frac{1}{2}\int_0^t B_s^T H_0 B_s ds \\
&= K_0^T U^{-1}\left[\int_0^t e^{\Lambda s} ds\right] Uu + \frac{1}{2}(Uu)^T\left[\int_0^t e^{\Lambda s}(U^{-1})^T H_0 U^{-1} e^{\Lambda s} ds\right] Uu, \quad \text{(A.23)}
\end{aligned}
$$

where we have used Equation (A.22).

Since we are only interested in the limit case when $T \to \infty$, and the elements of $\Lambda$ are negative, the only term with non-zero limit in the exponent of Equation (A.17) is the second term of $A_t$ in Equation (A.23). To compute the second term of $A_t$, we define

$$
H \equiv \int_0^t e^{\Lambda s}(U^{-1})^T H_0 U^{-1} e^{\Lambda s} ds.
$$

It is easy to show that the elements of $H$, denoted as $H_{ij}$ for $i, j = 1, 2, 3, 4$, can be computed as

$$
H_{ij} = -\frac{1}{\lambda_i + \lambda_j}[(U^{-1})^T H_0 U^{-1}]_{ij}. \quad \text{(A.24)}
$$

Then the covariance matrix C of $X_t$ and mean of $X_t$ can be written as

$$
C = U^T H U, \; m = -K_0^T U^{-1}\Lambda^{-1} U.
$$

In addition, the mean and covariance of $Y_t$ can be computed as in Equation (A.16). Then the regression coefficients in (A.14) can be readily computed. Moreover,

$$
\beta_0 = m e_1^T - \beta^T \mu, \qquad \sigma_u^2 = e_1 C e_1^T - \beta^T(\text{Var})\beta, \quad \text{(A.25)}
$$

where $\mu$ and Var are defined in (A.16).

Once the coefficients $\beta_i$'s are determined, by matching the coefficients of both sides of

$$
p_2\hat{\pi}_t + p_3\hat{\theta}_t = p_2\pi_t + p_3\theta_t = P_t - p_0 - p_1 D_t - p_4 A_t, \quad \text{(A.26)}
$$

we obtain

$$
\begin{aligned}
p_2\beta_0 + p_3\gamma_0 &= p_0, \\
p_2\beta_1 + p_3\gamma_1 &= -p_1, \\
p_2\beta_2 + p_3\gamma_2 &= 1, \\
p_2\beta_3 + p_3\gamma_3 &= -p_4.
\end{aligned}
$$

(A.27)

Hence, we find

$$\gamma_0 = \frac{-p_0 - p_2\beta_0}{p_3},\ \gamma_1 = \frac{-p_1 - p_2\beta_1}{p_3},\ \gamma_2 = \frac{1 - p_2\beta_2}{p_3},\ \gamma_3 = \frac{-p_4 - p_2\beta_3}{p_3}. \tag{A.28}$$

This accomplishes the task.

# D Proof of Lemma 2

Given $\hat{\pi}_t$ and $\hat{\theta}_t$ in (20) and (21), and the dynamics of $\Lambda_t$ in Equation (26), we obtain

$$\begin{aligned} dP_t &= p_1 dD_t + d\Lambda_t + p_4 dA_t \\ &= p_1(\hat{\pi}_t - \alpha_D D_t)dt + p_1\hat{\sigma}_D dB^u_{1t} + d\Lambda_t + p_4 dA_t \\ &= [q_0 + q_1 D_t + q_2 P_t + q_3 A_t]dt + p_1\hat{\sigma}_D dB^u_{1t} + \hat{\sigma}_\Lambda dB^u_{2t}, \end{aligned} \tag{A.29}$$

where

$$\begin{aligned} q_0 &= p_1\beta_0 + p_2\alpha_\pi(\bar{\pi} - \beta_0) - \alpha_\theta p_3\gamma_0, \\ q_1 &= p_1(\beta_1 - \alpha_D) - p_2\alpha_\pi\beta_1 - p_3\alpha_\theta\gamma_1, \\ q_2 &= p_1\beta_2 - p_2\alpha_\pi\beta_2 - p_3\alpha_\theta\gamma_2 + p_4, \\ q_3 &= p_1\beta_3 - p_2\alpha_\pi\beta_3 - p_3\alpha_\theta\gamma_3 - p_4\alpha. \end{aligned}$$

Applying further Equations (A.28), we obtain

$$\begin{aligned} q_0 &= p_1\beta_0 + p_2\alpha_\pi\bar{\pi} + p_0(\alpha_\pi + \alpha_\theta), \\ q_1 &= p_1(\beta_1 - \alpha_D + \alpha_\pi + \alpha_\theta), \\ q_2 &= p_1\beta_2 + (p_4 - \alpha_\pi - \alpha_\theta), \\ q_3 &= p_1\beta_3 + p_4(\alpha_\pi + \alpha_\theta - \alpha). \end{aligned} \tag{A.30}$$

The parameters $\hat{\sigma}^2_D$, $\hat{\sigma}^2_\Lambda$ and $\varrho$ are defined in Equations (24), (28) and (29), and $\sigma^2_u$ is defined in Equation (A.25).

Combined with Equation (22) and

$$dA_t = (P_t - \alpha A_t)dt, \tag{A.31}$$

we obtain the dynamics for $\Psi^u = (1, D_t, P_t, A_t)^T$, which follows the following SDE,

$$d\Psi^u = e^u_\psi \Psi^u dt + \sigma^u_\Psi dB^u_t, \tag{A.32}$$

where

$$e_{\psi}^{u} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \beta_0 & \beta_1 - \alpha_D & \beta_2 & \beta_3 \\ q_0 & q_1 & q_2 & q_3 \\ 0 & 0 & 1 & -\alpha \end{pmatrix}, \tag{A.33}$$

and

$$\sigma_{\psi}^{u} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \hat{\sigma}_D & 0 & 0 \\ 0 & p_1\hat{\sigma}_D + \varrho\hat{\sigma}_\Lambda & \sqrt{1-\varrho^2}\hat{\sigma}_\Lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A.34}$$

The investment opportunity is

$$dQ = (D_t - rP_t)dt + dP_t \equiv e_Q^u \Psi dt + \sigma_Q^u dB_t, \tag{A.35}$$

obtained via Equation (A.29), where $e_Q^u$ and $\sigma_Q^u$ are given by

$$e_Q^u = \begin{pmatrix} q_0 & 1+q_1 & q_2 - r & q_3 \end{pmatrix}, \tag{A.36}$$

and

$$\sigma_Q^u = \begin{pmatrix} 0 & p_1\hat{\sigma}_D + \varrho\hat{\sigma}_\Lambda & \sqrt{1-\varrho^2}\hat{\sigma}_\Lambda & 0 \end{pmatrix}. \tag{A.37}$$

Then we have the lemma. QED.

# E Proof of Proposition 2

To prove Proposition 2, we conjecture a solution for the portfolio demand by the un-informed investors as linear function of state variable set $\Psi^u$ as in Equation (37), and conjecture accordingly the value function

$$J^u(W^u, D_t, P_t, A_t; t) = -e^{-\rho t - rW - \frac{1}{2}\Psi^{uT}V^u\Psi^u}. \tag{A.38}$$

Substituting this into the HJB Equation (34), we obtain

$$\eta^u = f^u \Psi^u, \tag{A.39}$$

where

$$f^u = \frac{1}{r}(\sigma_Q\sigma_Q^T)^{-1}(e_Q - \sigma_Q\sigma_\Psi^T V^u) \tag{A.40}$$

with $V^u$ a symmetric positive definite matrix satisfying

$$V^u\sigma_\Psi\sigma_\Psi^T V^{uT} - (\sigma_Q\sigma_Q^T)^{-1}(e_Q - \sigma_Q\sigma_\Psi^T V^u)^T(e_Q - \sigma_Q\sigma_\Psi^T V^u) + rV^u - (e_\Psi^T V^u + V^u e_\Psi) + 2k\delta_{11}^{(4)} = 0, \tag{A.41}$$

$k \equiv [(r-\rho) - r\ln r] - \frac{1}{2}Tr(\sigma_\Psi^T\sigma_\Psi V^u)$ and

$$[\delta^{(4)}_{(11)}]_{ij} = \begin{cases} 1, & i=j=1 \\ 0, & \text{otherwise.} \end{cases} \quad \text{(A.42)}$$

This implies Proposition 2. QED.

# F Computation for the Autocovariance

We want to compute the autocovariance of the risky asset return defined as

$$\rho_\tau = \frac{1}{\tau^2} < P_{t+\tau} - P_t, P_t - P_{t-\tau} > .$$

It suffices to compute the autocovariance of $< P_{t+\tau}, P_t >$ for any $\tau > 0$. Note that the solution to the MA signal $A_t$, according to Equation (5), is

$$A_t = \int_{-\infty}^{t} e^{(p_4-\alpha)(t-s)}(p_0 + p_1 D_s + p_2\pi_s + p_3\theta_s)ds, \quad \text{(A.43)}$$

that is, $A_t$ is the moving average of the state variables $D_t$, $\pi_t$ and $\theta_t$ which are stationary. Note that in our model, we do not assume a priori any fixed correlation among the state variables, hence the autocovariance of can be compute as

$$< P_{t+\tau}, P_t > = < F^D_{t+\tau}, F^D_t > + < F^\pi_{t+\tau}, F^\pi_t > + < F^\theta_{t+\tau}, F^\theta_t >, \quad \text{(A.44)}$$

where

$$\begin{aligned} F^D_t &\equiv D_t + p_4\int_{-\infty}^{t} D_s e^{-\alpha_1(t-s)}ds, \\ F^\pi_t &\equiv \pi_t + p_4\int_{-\infty}^{t} \pi_s e^{-\alpha_1(t-s)}ds, \\ F^\theta_t &\equiv \theta_t + p_4\int_{-\infty}^{t} \theta_s e^{-\alpha_1(t-s)}ds, \end{aligned}$$

and $\alpha_1 = \alpha - p_4$. Hence, we have

$$\begin{aligned} & < F^D_{t+\tau}, F^D_t > \\ = \; & k_D\Big\{ e^{-\alpha_D\tau} + p_4[\frac{e^{-\alpha_D\tau}}{\alpha_1+\alpha_D} + \frac{e^{-\alpha_1\tau}}{\alpha_1+\alpha_D} + \frac{e^{-\alpha_1\tau} - e^{-\alpha_D\tau}}{\alpha_D - \alpha_1}] \\ & + p_4^2[\frac{e^{-\alpha_1\tau}}{(\alpha_D+\alpha_1)\alpha_1} + \frac{(e^{-\alpha_1\tau} - e^{-\alpha_D\tau})}{(\alpha_1+\alpha_D)(\alpha_D-\alpha_1)}]\Big\}, \end{aligned}$$

where $k_D = \sigma_D^2/2\alpha_D$. The formulas for $< F^{\pi}_{t+\tau}, F^{\pi}_t >$ and $< F^{\theta}_{t+\tau}, F^{\theta}_t >$ are similar with $\alpha_D$ ($\sigma_D$) replaced by $\alpha_\pi$ ($\sigma_\pi$) and $\alpha_\theta$ ($\sigma_\theta$), respectively. Now, let $g(\alpha, \tau) = -\frac{(1-e^{-\alpha\tau})^2}{\tau^2}$, we obtain

$$\rho_\tau = p_1^2[A_D g(\alpha_D, \tau) + B_D g(\alpha_1, \tau)] + p_2^2[A_\pi g(\alpha_\pi, \tau) + B_\pi g(\alpha_1, \tau)] + p_3^2[A_\theta g(\alpha_\theta, \tau) + B_\theta g(\alpha_1, \tau)], \tag{A.45}$$

where

$$\begin{aligned} A_D &= k_D \left[1 - \frac{p_4(2\alpha_1 + p_4)}{(\alpha_1 + \alpha_D)(\alpha_D - \alpha_1)}\right], \\ B_D &= k_D \frac{p_4(2\alpha_1 + p_4)}{(\alpha_1 + \alpha_D)(\alpha_D - \alpha_1)} \frac{\alpha_D}{\alpha_1}, \end{aligned}$$

and $A_\pi$,$A_\theta$,$B_\pi$, and $B_\theta$ are similarly defined.

**Table I**
**Stock Price versus Fraction of Technical Traders**

The table shows the impact of the fraction of technical traders, $w$, on the equilibrium stock price,

$$P_t = p + p_D D_t + p_\pi \pi_t + p_\theta \theta_t + p_{mv}(P_t - \alpha A_t).$$

The parameters are $r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.6, \sigma_\theta = 3.0, \alpha_\pi = 0.2, \alpha_\theta = 0.4, \alpha_D = 1.0$. The two panels present the results for two different moving average windows measured by $1/\alpha$ with $\alpha = 1$ and 0.1, respectively.

| **Panel A:** $\alpha = 1$ | | | | | |
|---|---|---|---|---|---|
| $w$ | $p$ | $p_D$ | $p_\pi$ | $p_\theta$ | $p_{mv}$ |
| 0 | 6.8209 | 0.9524 | 3.8095 | -1.0803 | 0.0000 |
| 0.1 | 8.9806 | 1.0148 | 3.6265 | -1.1449 | 0.0003 |
| 0.2 | 12.0439 | 1.0887 | 3.4124 | -1.2129 | 0.0002 |
| 0.3 | 16.0629 | 1.1758 | 3.1629 | -1.2841 | -0.0004 |
| 0.4 | 20.1075 | 1.2777 | 2.8731 | -1.3586 | -0.0016 |
| 0.5 | 21.6172 | 1.3971 | 2.5385 | -1.4362 | -0.0038 |
| 0.6 | 18.9471 | 1.5362 | 2.1533 | -1.5158 | -0.0070 |
| 0.7 | 14.2068 | 1.6978 | 1.7122 | -1.5968 | -0.0118 |
| 0.8 | 9.9461 | 1.8845 | 1.2094 | -1.6763 | -0.0182 |
| 0.9 | 6.8820 | 2.1002 | 0.6403 | -1.7516 | -0.0268 |
| 1 | 4.8052 | 2.3489 | 0.0000 | -1.8180 | -0.0382 |
| **Panel B:** $\alpha = 0.1$ | | | | | |
| 0 | 6.8209 | 0.9524 | 3.8095 | -1.0803 | 0.0000 |
| 0.1 | 8.6479 | 1.0225 | 3.6589 | -1.1644 | -0.0070 |
| 0.2 | 10.9427 | 1.1069 | 3.4747 | -1.2565 | -0.0152 |
| 0.3 | 13.4936 | 1.2093 | 3.2539 | -1.3589 | -0.0256 |
| 0.4 | 15.6019 | 1.3338 | 2.9918 | -1.4736 | -0.0395 |
| 0.5 | 16.2126 | 1.4825 | 2.6763 | -1.5996 | -0.0560 |
| 0.6 | 14.9448 | 1.6605 | 2.3011 | -1.7382 | -0.0764 |
| 0.7 | 12.5443 | 1.8719 | 1.8561 | -1.8883 | -0.1013 |
| 0.8 | 10.0112 | 2.1212 | 1.3313 | -2.0469 | -0.1312 |
| 0.9 | 7.8738 | 2.4191 | 0.7181 | -2.2140 | -0.1696 |
| 1 | 6.2033 | 2.7752 | 0.0000 | -2.3826 | -0.2180 |

## Table II
## Asset Allocation by Informed Investors versus Technical Traders

The table shows the allocations of the informed and technical investors,

$$\begin{aligned} \eta^i &= g_0^i + g_D^i D_t + g_\pi^i \pi_t + g_\theta^i \theta_t + g_{mv}^i MV_t, \\ \eta^u &= g_0^u + g_D^u D_t + g_\lambda^u \Lambda_t + g_{mv}^u MV_t, \end{aligned}$$

where $\eta^i$ and $\eta^u$ are demand by informed and technical investors, respectively, $MV_t = P_t - \alpha A_t$, and $\Lambda_t = P_t - (p - p_D D_t - p_{mv} MV_t)$. The parameters are $r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.6, \sigma_\theta = 3.0, \alpha_\pi = 0.2, \alpha_\theta = 0.4, \alpha_D = 1.0$, and $\alpha = 1$. Panel B shows the updating rule of Equation (18) and (19) used by technical traders.

**Panel A: Allocation Coefficients (g)**

| | Informed | | | | | Technical | | | |
|---|---|---|---|---|---|---|---|---|---|
| w | $g_0^i$ | $g_D^i$ | $g_\pi^i$ | $g_\theta^i$ | $g_{mv}^i$ | $g_0^u$ | $g_D^u$ | $g_\lambda^u$ | $g_{mv}^u$ |
| 0 | 1.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 4.2075 | 1.0010 | -0.4780 | 0.0007 |
| 0.1 | 0.6012 | -0.1062 | 0.1957 | 1.0493 | -0.0005 | 4.5892 | 0.9561 | -0.4855 | 0.0044 |
| 0.2 | 0.0278 | -0.2253 | 0.4184 | 1.1013 | -0.0004 | 4.8887 | 0.9011 | -0.4903 | 0.0014 |
| 0.3 | -0.7446 | -0.3583 | 0.6713 | 1.1560 | 0.0006 | 5.0706 | 0.8359 | -0.4955 | -0.0014 |
| 0.4 | -1.5700 | -0.5066 | 0.9582 | 1.2136 | 0.0026 | 4.8554 | 0.7599 | -0.5010 | -0.0039 |
| 0.5 | -1.9856 | -0.6719 | 1.2832 | 1.2740 | 0.0058 | 3.9855 | 0.6719 | -0.5074 | -0.0057 |
| 0.6 | -1.6424 | -0.8560 | 1.6516 | 1.3373 | 0.0104 | 2.7621 | 0.5707 | -0.5149 | -0.0070 |
| 0.7 | -0.8740 | -1.0618 | 2.0697 | 1.4032 | 0.0168 | 1.8028 | 0.4550 | -0.5242 | -0.0072 |
| 0.8 | -0.1508 | -1.2931 | 2.5460 | 1.4714 | 0.0251 | 1.2877 | 0.3233 | -0.5358 | -0.0063 |
| 0.9 | 0.3729 | -1.5559 | 3.0921 | 1.5410 | 0.0359 | 1.0697 | 0.1729 | -0.5510 | -0.0040 |
| 1 | 0.7192 | -1.8601 | 3.7258 | 1.6109 | 0.0498 | 1.0000 | 0.0000 | -0.5711 | 0.0000 |

**Panel B: Updating Rule by Technical Traders**

| w | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_3$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 7.1220 | 2.0024 | -0.6175 | -0.0010 | 0.2292 | 0.3178 | 0.0874 | -0.0003 |
| 0.1 | 8.6724 | 1.9929 | -0.6187 | -0.0064 | 0.2615 | 0.3493 | 0.0804 | -0.0019 |
| 0.2 | 10.6822 | 1.9652 | -0.6247 | -0.0021 | 0.2672 | 0.3794 | 0.0710 | -0.0007 |
| 0.3 | 13.1496 | 1.9249 | -0.6283 | 0.0022 | 0.2602 | 0.4098 | 0.0612 | 0.0008 |
| 0.4 | 15.2974 | 1.8712 | -0.6293 | 0.0065 | 0.2352 | 0.4401 | 0.0511 | 0.0025 |
| 0.5 | 15.4069 | 1.8036 | -0.6271 | 0.0105 | 0.2009 | 0.4700 | 0.0406 | 0.0044 |
| 0.6 | 12.7588 | 1.7223 | -0.6217 | 0.0140 | 0.1826 | 0.4991 | 0.0300 | 0.0066 |
| 0.7 | 9.1019 | 1.6282 | -0.6129 | 0.0169 | 0.1907 | 0.5269 | 0.0194 | 0.0089 |
| 0.8 | 6.0865 | 1.5232 | -0.6011 | 0.0191 | 0.2122 | 0.5529 | 0.0088 | 0.0114 |
| 0.9 | 4.0147 | 1.4099 | -0.5868 | 0.0204 | 0.2345 | 0.5768 | -0.0017 | 0.0141 |
| 1 | 2.6432 | 1.2921 | -0.5711 | 0.0210 | 0.2524 | 0.5986 | -0.0121 | 0.0169 |

## Table III
## The Trend Factor and Other Factors: Summary Statistics

This table reports the summary statistics of the trend factor (*Trend*), short-term reversal factor (*SREV*), momentum factor (*UMD*), long-term reversal factor (*LREV*), and Fama-French three factors including the market portfolio (*Market*), SMB and HML factors. We report the summary statistics such as sample mean in percentage, sample standard deviation in percentage, Sharpe ratio, skewness, and kurtosis. The sample period is from June 1930 to December 2013.

| Factor | Mean(%) | Std Dev(%) | Sharpe Ratio | Skewness | Kurtosis |
|---|---|---|---|---|---|
| Trend | 1.63 | 3.49 | 0.47 | 1.47 | 12.7 |
| SREV | 0.80 | 3.50 | 0.23 | 0.92 | 8.20 |
| UMD | 0.65 | 4.82 | 0.13 | -3.10 | 27.8 |
| LREV | 0.35 | 3.49 | 0.10 | 2.86 | 24.3 |
| Market | 0.62 | 5.40 | 0.11 | 0.23 | 7.62 |
| SMB | 0.28 | 3.25 | 0.09 | 2.13 | 21.3 |
| HML | 0.41 | 3.56 | 0.11 | 1.91 | 15.6 |

## Table IV
## The Trend Factor and Other Factors: Recession Periods

This table reports the summary statistics of the trend factor (*Trend*), short-term reversal factor (*SREV*), momentum factor (*UMD*), long-term reversal factor (*LREV*), and Fama-French three factors including the market portfolio (*Market*), SMB and HML factors. We report the summary statistics such as sample mean in percentage, sample standard deviation in percentage, Sharpe ratio, skewness, and kurtosis for the recession periods in Panel A, and for the most recent financial crisis period in Panel B. The sample period is from June 1930 to December 2013.

| Factor | Mean(%) | Std Dev(%) | Sharpe Ratio | Skewness | Kurtosis |
|---|---|---|---|---|---|
| | **Panel A: Recession Periods** | | | | |
| Trend | 2.30 | 4.99 | 0.46 | 0.89 | 5.76 |
| SREV | 1.15 | 5.40 | 0.21 | 0.78 | 3.38 |
| UMD | 0.40 | 8.14 | 0.05 | -2.91 | 14.8 |
| LREV | 0.47 | 4.11 | 0.11 | 1.32 | 7.02 |
| Market | -0.68 | 8.20 | -0.08 | 0.48 | 3.80 |
| SMB | 0.03 | 3.35 | 0.01 | 0.58 | 2.29 |
| HML | 0.16 | 4.94 | 0.03 | 2.65 | 16.8 |
| | **Panel B: Financial Crisis (12/2007 - 06/2009)** | | | | |
| Trend | 0.79 | 4.79 | 0.16 | 0.73 | 0.37 |
| SREV | -0.80 | 5.65 | -0.14 | -0.11 | -1.10 |
| UMD | -1.33 | 10.6 | -0.13 | -1.81 | 4.69 |
| LREV | 0.03 | 3.74 | 0.01 | 0.20 | -0.10 |
| Market | -2.03 | 7.07 | -0.29 | -0.21 | -0.24 |
| SMB | 0.63 | 2.50 | 0.25 | 0.26 | -0.76 |
| HML | -0.44 | 3.83 | -0.11 | -0.84 | 0.88 |

**Table V**

**The Trend Factor and Other Factors: Extreme Values and Correlation Matrix**

This table reports the maximum drawdown, Calmar ratio, and frequency of big losses of the trend factor (*Trend*), short-term reversal factor (*SREV*), momentum factor (*UMD*), long-term reversal factor (*LREV*), and the market portfolio (*Market*) in Panel A and the correlation matrix of the factors in Panel B, respectively. The sample period is from June 1930 to December 2013.

**Panel A: Extreme Values**

| Variable | MaxDrawDown(%) | Calmar Ratio(%) | N | $R < 0\%$ | $R < -10\%$ | $R < -20\%$ |
|---|---|---|---|---|---|---|
| Trend | 20.7 | 94.7 | 1,002 | 248 | 4 | 1 |
| SREV | 33.2 | 28.8 | 1,002 | 392 | 6 | 1 |
| UMD | 77.3 | 10.0 | 1,002 | 369 | 17 | 6 |
| LREV | 46.3 | 9.01 | 1,002 | 489 | 3 | 0 |
| Market | 76.6 | 9.67 | 1,002 | 404 | 30 | 5 |

**Panel B: Correlation Matrix**

| | Trend | SREV | UMD | LREV | Market |
|---|---|---|---|---|---|
| Trend | 1.00 | 0.33 | 0.00 | 0.13 | 0.20 |
| SREV | | 1.00 | -0.18 | 0.02 | 0.20 |
| UMD | | | 1.00 | -0.23 | -0.35 |
| LREV | | | | 1.00 | 0.26 |
| Market | | | | | 1.00 |

**Table VI**
**Comparison of Trend and Momentum**

This table compares the long and short portfolios of the trend factor and momentum factor. The summary statistics are reported for each of the long and short portfolios over the whole sample period (Panel A), the recession periods (Panel B) and the expansion periods (Panel C) identified by the NBER. A one-sided test of equal mean between the long (short) portfolios of the trend factor and momentum factor is reported in the table labeled as *Differ*. For the long portfolio, the test is $H_0 : \mu^l_{trd} = \mu^l_{mom}$; $H_1 : \mu^l_{trd} > \mu^l_{mom}$; for the short portfolio, the test is $H_0 : \mu^s_{trd} = \mu^s_{mom}$; $H_1 : \mu^s_{trd} < \mu^s_{mom}$, where the subscript $trd$ and $mom$ denote the trend and momentum factors, respectively; superscript $l$ and $s$ denote long and short portfolios, respectively. The last column (*Corr*) reports the correlation between the long (short) portfolios of the trend factor and momentum factor. Significance at the 1%, 5%, and 10% levels is given by an ***, and **, and an *, respectively. The sample period is from June 1930 to December 2013.

| Variable | Mean(%) | Std Dev(%) | Skewness | Kurtosis | N | Differ (%) | Corr |
|---|---|---|---|---|---|---|---|
| | **Panel A: Whole Sample Period** | | | | | | |
| Trend Long | 2.32 | 7.55 | 0.56 | 10.0 | 1,002 | 0.48*** | 0.89 |
| Momentum Long | 1.84 | 7.40 | 0.18 | 8.84 | 1,002 | | |
| Trend Short | 0.69 | 6.89 | 0.58 | 13.5 | 1,002 | -0.35** | 0.84 |
| Momentum Short | 1.04 | 11.3 | 2.91 | 25.7 | 1,002 | | |
| | **Panel B: Recession Periods** | | | | | | |
| Trend Long | 1.08 | 11.3 | 0.52 | 5.74 | 190 | 1.12*** | 0.87 |
| Momentum Long | -0.04 | 8.53 | -0.32 | 3.88 | 190 | | |
| Trend Short | -1.22 | 10.2 | 1.13 | 11.2 | 190 | -0.96* | 0.91 |
| Momentum Short | -0.27 | 16.6 | 2.31 | 15.3 | 190 | | |
| | **Panel C: Expansion Periods** | | | | | | |
| Trend Long | 2.61 | 6.34 | 0.79 | 11.7 | 812 | 0.34*** | 0.91 |
| Momentum Long | 2.27 | 7.04 | 0.47 | 10.9 | 812 | | |
| Trend Short | 1.13 | 5.77 | 0.27 | 9.95 | 812 | -0.21 | 0.80 |
| Momentum Short | 1.34 | 9.58 | 3.23 | 29.5 | 812 | | |

## Table VII
## The Spanning Tests

This table reports the results of testing whether the trend factor can be spanned by the short-term reversal, momentum, and long-term reversal factors. $W$ is the Wald test under conditional homoskedasticity, $W_e$ is the Wald test under the IID elliptical, $W_a$ is the Wald test under the conditional heteroskedasticity, $J_1$ is the Bekerart-Urias test with the Errors-in-Variables (EIV) adjustment, $J_2$ is the Bekerart-Urias test without the EIV adjustment and $J_3$ is the DeSantis test. All of the six tests have an asymptotic Chi-Squared distribution with $2N(N = 1)$ degrees of freedom. The $p$-values are in the brackets. The sample period is from June 1930 to December 2013.

| Period | W | $W_e$ | $W_a$ | $J_1$ | $J_2$ | $J_3$ |
|---|---|---|---|---|---|---|
| Whole Sample Period | 162.85 | 103.05 | 90.24 | 53.74 | 57.78 | 66.78 |
| | [0.00] | [0.00] | [0.00] | [0.00] | [0.00] | [0.00] |
| Recession Period | 17.48 | 12.72 | 15.77 | 16.28 | 18.03 | 19.91 |
| | [0.00] | [0.00] | [0.00] | [0.00] | [0.00] | [0.00] |
| Financial Crisis | 7.69 | 6.97 | 10.52 | 6.44 | 6.33 | 11.78 |
| | [0.02] | [0.03] | [0.01] | [0.04] | [0.04] | [0.00] |

## Table VIII
## Average Returns and Other Characteristics of the Trend Quintile Portfolios

This table reports the average return and other characteristics of the five trend quintile portfolios. Market size is in millions of dollars. $R_{-1}$(%). $R_{-6,-2}$(%) and $R_{-60,-2}$(%) are prior month return, past six-month cumulative return skipping the last month, past 60-month cumulative return skipping the last month, respectively. Idio Vol(%) is the idiosyncratic volatility relative to the Fama-French three-factor model estimated from the daily returns of each month. %Zero is the percentage of zero returns in a month. Turnover(%) is the monthly turnover rate of the stocks. E/P and S/P are earnings-price ratio and sales-price ratio, respectively. Newey and West (1987) robust $t$-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by an ***, and **, and an *, respectively. The sample period is from June 1930 to December 2013.

| Rank | Return(%) | Market Size | $log$B/M | $R_{-1}$(%) | $R_{-6,-2}$(%) | $R_{-60,-2}$(%) | Idio Vol(%) | %Zero | Turnover(%) | E/P | S/P |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Low | 0.69*** | 1,154.0*** | -1.11*** | 7.50*** | 6.46*** | 98.1*** | 2.18*** | 15.7*** | 6.60*** | 1.65*** | 76.7*** |
| | (2.94) | (8.45) | (-9.69) | (15.9) | (5.26) | (24.4) | (31.8) | (25.9) | (13.8) | (5.06) | (11.3) |
| 2 | 1.09*** | 1,717.2*** | -1.07*** | 3.29*** | 6.79*** | 86.5*** | 1.77*** | 16.1*** | 5.19*** | 4.12*** | 87.9*** |
| | (5.14) | (8.39) | (-8.47) | (12.9) | (6.99) | (26.2) | (40.4) | (26.3) | (13.2) | (10.5) | (13.2) |
| 3 | 1.31*** | 1,931.1*** | -1.04*** | 1.37*** | 7.51*** | 83.8*** | 1.69*** | 16.4*** | 4.84*** | 4.68*** | 91.9*** |
| | (6.47) | (8.23) | (-8.19) | (6.97) | (8.27) | (26.6) | (41.6) | (26.2) | (12.9) | (11.0) | (13.5) |
| 4 | 1.61*** | 1,906.4*** | -1.04*** | -0.44** | 8.47*** | 85.5*** | 1.77*** | 16.3*** | 4.96*** | 4.54*** | 96.1*** |
| | (7.73) | (8.04) | (-8.32) | (-2.23) | (9.14) | (26.6) | (39.9) | (25.9) | (13.1) | (11.3) | (13.4) |
| High | 2.32*** | 1,404.3*** | -1.09*** | -3.47*** | 10.6*** | 97.9*** | 2.19*** | 16.1*** | 6.15*** | 2.17*** | 90.0*** |
| | (9.66) | (7.95) | (-9.13) | (-12.5) | (9.38) | (26.3) | (33.7) | (26.1) | (13.1) | (5.05) | (13.0) |

## Table IX
## CAPM and Fama-French Alphas

This table reports Jensen's alpha and risk loadings with respect to the CAPM and Fama-French three-factor model, respectively, for the five trend quintile portfolios, the trend factor, and the UMD factor. The alphas are reported in percentage. Newey and West (1987) robust $t$-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by an ***, and **, and an *, respectively. The sample period is from June 1930 to December 2013.

| | **Panel A: CAPM** | | **Panel B: Fama-French** | | | |
|---|---|---|---|---|---|---|
| Rank | $\alpha(\%)$ | $\beta_{mkt}$ | $\alpha(\%)$ | $\beta_{mkt}$ | $\beta_{smb}$ | $\beta_{hml}$ |
| Low | -0.32*** | 1.17*** | -0.46*** | 1.02*** | 0.60*** | 0.15*** |
| | (-3.49) | (41.6) | (-6.29) | (41.6) | (11.6) | (4.12) |
| 2 | 0.13* | 1.08*** | 0.01 | 0.96*** | 0.42*** | 0.18*** |
| | (1.95) | (52.6) | (0.32) | (72.1) | (9.24) | (4.73) |
| 3 | 0.37*** | 1.06*** | 0.25*** | 0.96*** | 0.36*** | 0.19*** |
| | (5.25) | (42.5) | (5.87) | (46.6) | (10.6) | (3.86) |
| 4 | 0.63*** | 1.12*** | 0.51*** | 1.01*** | 0.38*** | 0.20*** |
| | (8.73) | (37.9) | (11.0) | (41.1) | (8.18) | (4.05) |
| High | 1.23*** | 1.30*** | 1.09*** | 1.14*** | 0.61*** | 0.16*** |
| | (12.6) | (52.8) | (16.3) | (59.7) | (10.7) | (4.14) |
| High-Low | 1.56*** | 0.13*** | 1.55*** | 0.12*** | 0.00 | 0.01 |
| (Trend) | (13.7) | (3.23) | (13.1) | (3.31) | (0.03) | (0.21) |
| UMD | 0.88*** | -0.30*** | 1.02*** | -0.22*** | -0.05 | -0.45*** |
| | (7.24) | (-3.09) | (8.31) | (-3.50) | (-0.59) | (-3.55) |

**Table X**
**Robustness of the Trend Factor**

This table reports the summary statistics of the various specifications of the trend factor. *Size Filter*: stocks in the smallest decile based on the NYSE size breakpoints are excluded. *Price Filter*: Stocks whose prices are less than $5 are excluded. *No Filter*: No size restriction nor price restriction is imposed. *Fama-French*: the trend factor is constructed following Fama-French approach; stocks are first independently sorted into two size groups and three trend groups using NYSE stocks only, and then averaged across the two size groups. The summary statistics are sample mean in percentage, sample standard deviation in percentage, Sharpe ratio, skewness, and kurtosis. The sample period is from June 1930 to December 2013.

| Specification | Mean(%) | Std Dev(%) | Sharpe Ratio | Skewness | Kurtosis |
|---|---|---|---|---|---|
| No Filter | 2.92 | 4.54 | 0.64 | 2.06 | 16.3 |
| Size Filter | 1.97 | 4.12 | 0.48 | 2.11 | 16.8 |
| Price Filter | 1.80 | 3.38 | 0.53 | 1.45 | 11.1 |
| Fama-French | 1.91 | 3.20 | 0.60 | 2.42 | 15.0 |

**Table XI**
**The Trend Factor in Other G7 Countries**

This table compares the summary statistics of the trend factor (*Trend*), short-term reversal factor (*SREV*), momentum factor (*UMD*) and long-term reversal factor (*LREV*) in the other G7 countries. We report the summary statistics such as sample mean in percentage, sample standard deviation in percentage, Sharpe ratio, skewness, kurtosis as well as the CAPM alpha relative to the global market portfolio. Newey and West (1987) robust $t$-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by an ***, and **, and an *, respectively. The sample period is from January 1990 to December 2013.

| Variable | Mean(%) | Std Dev(%) | Sharpe Ratio | Skewness | Kurtosis | Alpha |
|---|---|---|---|---|---|---|
| | | | France | | | |
| Trend | 1.22*** | 5.69 | 0.21 | 1.91 | 12.5 | 0.97** |
| SREV | 0.64* | 6.04 | 0.11 | 0.27 | 5.97 | 0.38 |
| UMD | 1.81*** | 6.39 | 0.28 | -0.24 | 3.20 | 1.82*** |
| LREV | -0.34 | 5.10 | -0.07 | 0.86 | 5.47 | -0.43 |
| | | | United Kingdom | | | |
| Trend | 0.73*** | 3.68 | 0.20 | -0.03 | 1.92 | 0.86*** |
| SREV | 0.09 | 4.22 | 0.02 | -0.26 | 2.88 | 0.05 |
| UMD | 1.22*** | 5.55 | 0.22 | -2.10 | 15.6 | 1.06** |
| LREV | 0.56** | 3.42 | 0.16 | 0.67 | 1.16 | 0.63*** |
| | | | Germany | | | |
| Trend | 1.19*** | 5.83 | 0.20 | 0.72 | 7.12 | 1.07*** |
| SREV | 0.22 | 6.51 | 0.03 | 0.76 | 7.90 | 0.41 |
| UMD | 1.91*** | 7.48 | 0.26 | 0.06 | 7.97 | 1.60*** |
| LREV | 0.16 | 4.89 | 0.03 | 0.88 | 3.84 | 0.20 |
| | | | Italy | | | |
| Trend | 2.07*** | 7.92 | 0.26 | 4.73 | 35.8 | 1.54*** |
| SREV | 0.33 | 5.13 | 0.06 | -0.75 | 4.54 | -0.35 |
| UMD | 0.44 | 10.7 | 0.04 | -5.88 | 64.9 | 0.78 |
| LREV | 0.28 | 9.53 | 0.03 | 9.39 | 110.5 | 0.28 |
| | | | Canada | | | |
| Trend | 0.95*** | 5.37 | 0.18 | 0.22 | 1.51 | 0.82** |
| SREV | -0.08 | 6.39 | -0.01 | -0.83 | 6.18 | -0.06 |
| UMD | 1.89*** | 6.95 | 0.27 | -0.18 | 4.70 | 1.41*** |
| LREV | 0.48 | 5.15 | 0.09 | 0.18 | 1.31 | 0.61* |
| | | | Japan | | | |
| Trend | 0.99*** | 4.18 | 0.24 | -0.00 | 2.26 | 0.66*** |
| SREV | 0.78*** | 4.34 | 0.18 | 1.16 | 4.83 | 0.37 |
| UMD | 0.37 | 5.35 | 0.07 | -0.01 | 2.35 | 0.49 |
| LREV | 0.78*** | 3.34 | 0.23 | -0.03 | 1.04 | 0.82*** |

## Table XII
## The Trend Factor in Other G7 Countries During Financial Crisis

This table compares the summary statistics of the trend factor (*Trend*), short-term reversal factor (*SREV*), momentum factor (*UMD*) and long-term reversal factor (*LREV*) in the other G7 countries during the most recent financial crisis (December 2007 to June 2009). We report the summary statistics such as sample mean in percentage, sample standard deviation in percentage, Sharpe ratio, skewness, kurtosis as well as the CAPM alpha relative to the global market portfolio. Newey and West (1987) robust $t$-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by an ***, and **, and an *, respectively.

| Variable | Mean(%) | Std Dev(%) | Sharpe Ratio | Skewness | Kurtosis | Alpha |
|---|---|---|---|---|---|---|
| | | | France | | | |
| Trend | 1.34* | 3.40 | 0.39 | 0.08 | -0.97 | 0.96 |
| SREV | 1.23 | 4.09 | 0.30 | 0.96 | 1.55 | 1.52 |
| UMD | 0.48 | 7.20 | 0.07 | -1.02 | 1.56 | -0.88 |
| LREV | -0.39 | 3.70 | -0.11 | 0.45 | -1.08 | -0.05 |
| | | | United Kingdom | | | |
| Trend | 1.43 | 6.30 | 0.23 | -0.37 | -1.27 | 1.49 |
| SREV | -0.15 | 6.62 | -0.02 | -0.38 | 0.23 | 0.63 |
| UMD | -2.23 | 11.5 | -0.19 | -2.29 | 7.34 | -3.61 |
| LREV | 0.90 | 4.66 | 0.19 | 0.38 | 0.37 | 0.32 |
| | | | Germany | | | |
| Trend | 1.23 | 5.79 | 0.21 | 0.58 | 1.88 | 0.94 |
| SREV | 1.25 | 4.24 | 0.29 | -0.03 | 0.17 | 1.04 |
| UMD | 0.69 | 9.80 | 0.07 | -1.15 | 2.22 | -1.31 |
| LREV | 0.06 | 3.08 | 0.02 | -0.48 | -0.53 | -0.07 |
| | | | Italy | | | |
| Trend | 0.96 | 6.52 | 0.15 | 0.51 | 1.28 | 0.37 |
| SREV | 0.84 | 6.70 | 0.13 | 0.27 | 0.02 | 2.22** |
| UMD | 0.30 | 9.74 | 0.03 | -0.72 | 1.17 | -1.44 |
| LREV | -0.13 | 4.94 | -0.03 | -0.57 | 0.11 | -0.14 |
| | | | Canada | | | |
| Trend | 2.88 | 7.92 | 0.36 | 0.85 | -0.01 | 3.98** |
| SREV | -0.51 | 6.49 | -0.08 | -0.65 | 1.25 | 0.04 |
| UMD | -3.82 | 10.5 | -0.36 | -1.16 | 4.49 | -4.91** |
| LREV | 0.71 | 7.03 | 0.10 | 0.21 | -0.74 | 0.02 |
| | | | Japan | | | |
| Trend | 0.16 | 3.59 | 0.04 | -0.14 | 0.78 | 0.38 |
| SREV | 0.12 | 4.82 | 0.02 | 0.74 | 0.39 | 0.33 |
| UMD | -2.44 | 6.92 | -0.35 | -0.83 | 0.66 | -3.89*** |
| LREV | 1.00** | 1.83 | 0.55 | -0.65 | -0.14 | 0.74* |

## Table XIII
## Explaining Short-Term Reversal

This table reports the pricing ability of the trend factor and the momentum factor using the 10 short-term reversal decile portfolios (*STRev*). Panel A is the CAPM result; Panel B and C include the trend factor and the momentum factor, respectively. The intercept ($\alpha$) is in percentage. $\beta_{mkt}$, $\beta_{trd}$, and $\beta_{umd}$ are the risk loadings on the market portfolio, the trend factor, and the momentum factor, respectively. The $t$-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by an ***, and **, and an *, respectively. The second to the last column is the average pricing error $\Delta = \alpha'\Sigma^{-1}\alpha$. The last column is the GRS test statistics with their $p$-values in the brackets. The sample period is from June 1930 to December 2013.

| | STRev1 | STRev2 | STRev3 | STRev4 | STRev5 | STRev6 | STRev7 | STRev8 | STRev9 | STRev10 | $\Delta$ | GRS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Panel A: CAPM** | | | | | | | | | | | | |
| $\alpha(\%)$ | 1.61*** | 0.50*** | 0.32*** | 0.28*** | 0.19 | 0.23*** | 0.12 | 0.02 | -0.24** | -0.99*** | 0.28 | 26.9*** |
| | (7.58) | (4.08) | (2.98) | (2.91) | (1.18) | (2.71) | (1.31) | (0.22) | (-2.31) | (-7.19) | | [0.00] |
| $\beta_{mkt}$ | 1.59*** | 1.35*** | 1.27*** | 1.16*** | 1.02*** | 1.11*** | 1.13*** | 1.17*** | 1.20*** | 1.26*** | | |
| | (40.77) | (59.90) | (63.54) | (66.09) | (33.85) | (72.22) | (68.20) | (67.26) | (63.47) | (49.84) | | |
| **Panel B: CAPM plus Trend Factor** | | | | | | | | | | | | |
| $\alpha(\%)$ | 1.20*** | 0.22* | 0.10 | 0.17* | 0.39** | 0.26*** | 0.19* | 0.14 | 0.04 | -0.66*** | 0.23 | 18.5*** |
| | (5.19) | (1.67) | (0.82) | (1.66) | (2.15) | (2.85) | (1.95) | (1.31) | (0.35) | (-4.42) | | [0.00] |
| $\beta_{mkt}$ | 1.56*** | 1.33*** | 1.25*** | 1.15*** | 1.03*** | 1.12*** | 1.14*** | 1.18*** | 1.22*** | 1.29*** | | |
| | (39.51) | (58.51) | (62.08) | (64.54) | (33.82) | (71.04) | (67.35) | (66.72) | (64.57) | (50.65) | | |
| $\beta_{trd}$ | 0.27*** | 0.18*** | 0.15*** | 0.07** | -0.12*** | -0.02 | -0.05* | -0.07*** | -0.18*** | -0.21*** | | |
| | (4.35) | (5.11) | (4.67) | (2.43) | (-2.61) | (-0.93) | (-1.82) | (-2.68) | (-6.07) | (-5.42) | | |
| **Panel C: CAPM plus Momentum Factor** | | | | | | | | | | | | |
| $\alpha(\%)$ | 2.06*** | 0.74*** | 0.51*** | 0.41*** | 0.14 | 0.35*** | 0.24*** | 0.15 | -0.10 | -0.86*** | 0.31 | 29.7*** |
| | (10.19) | (6.32) | (4.81) | (4.35) | (0.84) | (4.28) | (2.73) | (1.62) | (-1.00) | (-6.26) | | [0.00] |
| $\beta_{mkt}$ | 1.43*** | 1.26*** | 1.20*** | 1.11*** | 1.04*** | 1.07*** | 1.08*** | 1.12*** | 1.15*** | 1.22*** | | |
| | (36.60) | (55.56) | (58.78) | (60.96) | (32.35) | (66.98) | (62.99) | (62.08) | (58.41) | (45.48) | | |
| $\beta_{umd}$ | -0.53*** | -0.29*** | -0.22*** | -0.16*** | 0.06* | -0.15*** | -0.15*** | -0.15*** | -0.16*** | -0.15*** | | |
| | (-12.13) | (-11.31) | (-9.63) | (-7.72) | (1.76) | (-8.41) | (-7.72) | (-7.66) | (-7.33) | (-4.96) | | |

## Table XIV
## Explaining Short-Term Reversal, Momentum, and Long-Term Reversal

This table compares the pricing ability of the trend factor and the momentum factor using the triple sorted portfolios. Stocks are independently sorted by the last month returns ($t-1$), the past 12-month cumulative returns skipping the last month (from $t-2$ to $t-12$), and the past 60-month cumulative returns from month $t-25$ and $t-60$ into $3\times 3\times 3$ groups and then in each group stocks are equal-weighted to form the portfolio. Panel A is the CAPM result; Panel B and C include the trend factor and the momentum factor, respectively. The intercept ($\alpha$) is in percentage. $\beta_{mkt}$, $\beta_{umd}$, and $\beta_{trd}$ are the risk loadings on the market portfolio, the momentum factor, and the trend factor, respectively. The $t$-statistics are in parentheses and significance at the 1%, 5%, and 10% levels is given by an ***, and **, and an *, respectively. The second to the last column is the average pricing error $\Delta = \alpha'\Sigma^{-1}\alpha$. The last column is the GRS test statistics with their $p$-values in the brackets. The sample period is from June 1930 to December 2013.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Panel A: CAPM** | | | | | | | | | | | | | | | |
| $\alpha(\%)$ | 0.89*** | 0.79*** | 0.55*** | 0.95*** | 0.81*** | 0.60*** | 1.13*** | 1.13*** | 1.02*** | 0.30** | 0.16 | -0.01 | 0.48*** | 0.38*** | 0.14* |
| | (6.24) | (6.02) | (4.22) | (6.24) | (9.09) | (6.13) | (9.01) | (10.16) | (8.25) | (2.48) | (1.60) | (-0.14) | (5.72) | (5.43) | (1.76) |
| $\beta_{mkt}$ | 1.29*** | 1.29*** | 1.36*** | 1.27*** | 1.10*** | 1.23*** | 1.25*** | 1.17*** | 1.31*** | 1.20*** | 1.15*** | 1.24*** | 0.99*** | 0.98*** | 1.10*** |
| | (49.01) | (53.58) | (56.57) | (45.30) | (66.68) | (68.26) | (53.62) | (56.88) | (57.10) | (53.50) | (61.34) | (65.09) | (64.11) | (76.52) | (78.20) |
| **Panel B: CAPM plus Trend Factor** | | | | | | | | | | | | | | | |
| $\alpha(\%)$ | 0.54*** | 0.71*** | 0.36** | 0.76*** | 0.55*** | 0.43*** | 0.84*** | 1.01*** | 0.66*** | 0.33** | 0.15 | 0.14 | 0.36*** | 0.28*** | 0.14* |
| | (3.51) | (4.94) | (2.49) | (4.60) | (5.75) | (4.04) | (6.16) | (8.29) | (4.94) | (2.45) | (1.32) | (1.24) | (3.99) | (3.68) | (1.68) |
| $\beta_{mkt}$ | 1.26*** | 1.29*** | 1.35*** | 1.25*** | 1.08*** | 1.21*** | 1.22*** | 1.16*** | 1.28*** | 1.21*** | 1.15*** | 1.26*** | 0.98*** | 0.97*** | 1.10*** |
| | (47.71) | (52.33) | (55.14) | (44.06) | (65.51) | (66.69) | (52.29) | (55.48) | (55.93) | (52.56) | (60.10) | (64.83) | (62.58) | (74.80) | (76.73) |
| $\beta_{trd}$ | 0.22*** | 0.05 | 0.12*** | 0.12*** | 0.17*** | 0.11*** | 0.19*** | 0.08** | 0.23*** | -0.02 | 0.01 | -0.10*** | 0.07*** | 0.06*** | -0.00 |
| | (5.45) | (1.33) | (3.32) | (2.68) | (6.55) | (3.83) | (5.21) | (2.39) | (6.62) | (-0.46) | (0.34) | (-3.31) | (3.01) | (3.14) | (-0.18) |
| **Panel C: CAPM plus Momentum Factor** | | | | | | | | | | | | | | | |
| $\alpha(\%)$ | 1.31*** | 1.26*** | 0.93*** | 1.17*** | 0.89*** | 0.70*** | 0.98*** | 1.01*** | 0.84*** | 0.72*** | 0.50*** | 0.33*** | 0.52*** | 0.45*** | 0.16** |
| | (10.62) | (12.35) | (8.26) | (7.92) | (9.97) | (7.20) | (7.84) | (9.12) | (6.92) | (7.43) | (6.05) | (3.93) | (6.16) | (6.54) | (2.05) |
| $\beta_{mkt}$ | 1.13*** | 1.12*** | 1.22*** | 1.18*** | 1.07*** | 1.19*** | 1.31*** | 1.21*** | 1.38*** | 1.04*** | 1.02*** | 1.11*** | 0.97*** | 0.95*** | 1.09*** |
| | (47.41) | (56.58) | (55.69) | (41.16) | (61.69) | (63.21) | (54.10) | (56.46) | (58.47) | (55.37) | (63.72) | (68.16) | (59.48) | (71.12) | (72.95) |
| $\beta_{umd}$ | -0.51*** | -0.58*** | -0.47*** | -0.28*** | -0.10*** | -0.13*** | 0.19*** | 0.14*** | 0.22*** | -0.52*** | -0.42*** | -0.43*** | -0.05*** | -0.09*** | -0.03* |
| | (-19.35) | (-26.28) | (-19.23) | (-8.74) | (-5.16) | (-6.03) | (7.14) | (6.02) | (8.50) | (-24.61) | (-23.29) | (-23.36) | (-2.88) | (-6.18) | (-1.77) |

| | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | $\Delta$ | GRS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Panel A: CAPM** | | | | | | | | | | | | | | |
| $\alpha(\%)$ | 0.81*** | 0.66*** | 0.70*** | -0.48*** | -0.49*** | -0.61*** | 0.07 | -0.14* | -0.01 | 0.67*** | 0.32*** | 0.43*** | 0.35 | 12.5*** |
| | (7.75) | (6.72) | (6.99) | (-4.06) | (-4.64) | (-5.73) | (0.74) | (-1.73) | (-0.13) | (5.42) | (2.85) | (3.50) | | [0.00] |
| $\beta_{mkt}$ | 1.06*** | 1.02*** | 1.15*** | 1.14*** | 1.12*** | 1.25*** | 1.05*** | 0.99*** | 1.14*** | 1.10*** | 1.03*** | 1.20*** | | |
| | (55.17) | (56.18) | (62.72) | (51.85) | (57.67) | (63.44) | (62.08) | (67.29) | (70.62) | (47.97) | (49.30) | (53.43) | | |
| **Panel B: CAPM plus Trend Factor** | | | | | | | | | | | | | | |
| $\alpha(\%)$ | 0.63*** | 0.41*** | 0.62*** | -0.20 | -0.14 | -0.28** | 0.08 | -0.01 | 0.08 | 0.52*** | 0.32** | 0.30** | 0.21 | 6.57*** |
| | (5.51) | (3.89) | (5.70) | (-1.56) | (-1.24) | (-2.47) | (0.82) | (-0.07) | (0.80) | (3.84) | (2.57) | (2.24) | | [0.00] |
| $\beta_{mkt}$ | 1.05*** | 1.00*** | 1.15*** | 1.17*** | 1.15*** | 1.28*** | 1.05*** | 1.00*** | 1.15*** | 1.09*** | 1.03*** | 1.19*** | | |
| | (53.75) | (54.87) | (61.28) | (52.61) | (59.62) | (65.09) | (60.95) | (67.17) | (69.88) | (46.67) | (48.33) | (52.08) | | |
| $\beta_{trd}$ | 0.12*** | 0.16*** | 0.05 | -0.18*** | -0.22*** | -0.21*** | -0.01 | -0.08*** | -0.06** | 0.10*** | 0.00 | 0.08** | | |
| | (3.87) | (5.63) | (1.62) | (-5.30) | (-7.51) | (-6.95) | (-0.36) | (-3.68) | (-2.24) | (2.68) | (0.06) | (2.31) | | |
| **Panel C: CAPM plus Momentum Factor** | | | | | | | | | | | | | | |
| $\alpha(\%)$ | 0.60*** | 0.43*** | 0.48*** | -0.14 | -0.18** | -0.31*** | 0.10 | -0.13 | -0.01 | 0.39*** | 0.00 | 0.14 | 0.33 | 11.6*** |
| | (6.04) | (4.74) | (5.12) | (-1.38) | (-1.98) | (-3.32) | (1.08) | (-1.59) | (-0.14) | (3.38) | (0.03) | (1.25) | | [0.00] |
| $\beta_{mkt}$ | 1.14*** | 1.10*** | 1.24*** | 1.02*** | 1.00*** | 1.14*** | 1.04*** | 0.99*** | 1.14*** | 1.20*** | 1.15*** | 1.31*** | | |
| | (59.80) | (63.07) | (68.62) | (50.15) | (56.90) | (62.61) | (57.73) | (62.99) | (66.35) | (54.05) | (59.87) | (60.18) | | |
| $\beta_{umd}$ | 0.26*** | 0.28*** | 0.27*** | -0.42*** | -0.38*** | -0.37*** | -0.04** | -0.01 | 0.00 | 0.34*** | 0.39*** | 0.35*** | | |
| | (12.23) | (14.48) | (13.38) | (-18.47) | (-19.22) | (-18.20) | (-1.99) | (-0.66) | (0.06) | (13.90) | (18.30) | (14.44) | | |

**Figure 1**
**Autocovariance of Asset Return**

This figure shows the autocovariance of asset return defined as

$$< \frac{P_{t+\tau} - P_t}{\tau}, \frac{P_t - P_{t-\tau}}{\tau} >,$$

where $\tau$ is the investment horizon. The autocovariance is positive for short horizon $\tau < 1$ and becomes negative over longer horizon. The parameters are $r = 0.05, \rho = 0.2, \bar{\pi} = 0.85, \sigma_D = 1.0, \sigma_\pi = 0.6, \sigma_\theta = 3.0, \alpha_\pi = 0.2, \alpha_\theta = 0.4, \alpha_D = 1, \alpha = 1.1$ The moving average window is measured by $1/\alpha$. The fraction of technical traders is $w = 0.2$.