Non Normal Disturbances

ϵ has $E(\epsilon) = 0$
and $Cov(\epsilon) = \sigma^2 I_T$,
but not Normal. [other assumptions hold.]

Small sample properties of OLS estimators :

$\hat{\beta}$ unbiased, BLUE

$Cov(\hat{\beta}) = \sigma^2 (X'X)^{-1}$

s^2 unbiased

— All proven, w/o Normality assumption.

Small sample Problems :

(i) There may be more efficient estimators than $\hat{\beta}$ (but not linear ones).
(ii) $\hat{\beta}$ is not $\sim$ Normal
(iii) s^2 is not $\sim$ χ^2 } (proofs need Normality assumptions)

Thus the former t-statistics are not $\sim$ t
& the former F-statistics are not $\sim$ F.

$\therefore$ immediate problems in how to test hypotheses!

Use Chebyschev Theorem to test hypothesis.

Can still use Chebyschev Theorem to make statements like

$$P(|\hat{\beta}-\beta| < 2\sigma) \geq .75,$$

but to even do this, we must estimate σ with s.
∴ Not cool from Purists' point of view.

All we can really do is compare each estimate with its standard error, and wave our hands.

Large Sample Properties:

$\hat{\beta}$ & s^2 consistent.

$\hat{\beta}$ is asymptotically Normal,

because $\hat{\beta} = \beta + (X'X)^{-1}X'\epsilon$
is a weighted sum of ϵ's,
which → a Normal Cumulative Distrib. by the C.L.T.

Note: $$\frac{\hat{\beta}_i - \beta_i^*}{\sqrt{s^2(X'X)^{-1}_{ii}}} \longrightarrow N(0,1) \quad \text{if } s^2 \to \sigma^2$$
(and it does!)

This solves our distributional problems;
the calculations that were our t-statistics
are asymptotically Normal.
Thus, t-tests and F-tests work asymptotically.
∴ A large sample serves as a substitute for the Normality ass.

How big is "big"?

Typically 50-100 observations are sufficient;
it depends upon how quickly $\hat{\beta}$ becomes Normal;
which in turn depends upon
how close the ϵ's are to being Normal.

i.e. Depends upon how the ϵ's are actually distributed

DIGRESSION:

$\exists$ a set of distributions with a characteristic function containing a parameter, θ, which indicates the following.

If $\theta = 1$; Cauchy Distribution (no moments exist)
If $\theta = 2$; Normal Distribution (all moments exist)
If $1 < \theta < 2$; "Symmetric Stable Distributions" (one moment exists, variance is infinite)

<u>Note:</u> Only infinite tailed distributions may not have moments.
If a distribution is truncated, it has moments; (the variance exists).
--- If the variance is finite, the C.L.T. applies, & large sample properties still hold.

Since $\nexists$ no economically interesting distributions that are not truncated,
we can apply asymptotics!