

3. VIOLATIONS OF THE IDEAL CONDITIONS

This ends our discussion of the Classical Regression Model under the ideal conditions.

∴ End of Statistician's part of the course.

Now consider Econometrician's part - Violations.

In problems that economists face, ^{in Accounting & Finance} ideal conditions often do not hold.

∴ The nice properties of OLS estimates are brought into question.

(i)
Not only are the hypothesis tests often invalid (since the distributions of the OLS estimates depend on the ideal conditions),

(ii)
but there are other ways to estimate the model that will be better than OLS.

Restrictions

We can sometimes impose restrictions on our model if we know they are true.

This is done to gain efficiency in estimation
→ to get better estimates.

Consider Exact Linear Restrictions

$$Y = X\beta + \epsilon \quad \text{with ideal conditions holding}$$

$$\text{Restrictions: } R\beta = r$$

$(m \times K) \quad (K \times 1) \quad m \times 1$

Consider the "restricted estimator",

$$\tilde{\beta} = \hat{\beta} + (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(r - R\hat{\beta})$$

where $\hat{\beta} = (X'X)^{-1}X'Y = \text{OLS estimate}$.

Properties of $\tilde{\beta}$:

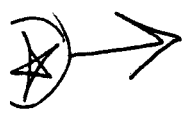
- 1) $\tilde{\beta}$ maximizes the likelihood function (and equivalently minimizes SSE) s.t. the restrictions, $R\beta = r$.
- 2) $R\tilde{\beta} = r$ [see EXAM] do on bd! (easy)
- 3) $\tilde{\beta}$ is efficient relative to $\hat{\beta}$; $[\text{Cov}(\hat{\beta}) - \text{Cov}(\tilde{\beta})]$ is ps.

under H₀:
 $R\beta = r$

recall Lagrange Multiplier
First Order Conditions
Max π s.t. budget constraint
max utility s.t. income constraint

~~ANOTHER RESTRICTIONS, R\beta=r~~

☆ This 3rd property is true; $[Cov(\hat{\beta}) - Cov(\tilde{\beta})]$ is psd.;
 whether or not the imposed restrictions, $R\beta=r$, are true.
 Hence, imposing the restrictions reduces the variance,
 though it will introduce bias if the restrictions
 are not true.



Intuition: By imposing restrictions,
 you constrain the model;
 you reduce the number of things in the model
 that can vary.
 $\Rightarrow \therefore$ You reduce the variability of the model.

But if $R\beta \neq r$ in reality,
 then $\tilde{\beta} = \hat{\beta} + [\text{something with Expected Value} \neq 0]$.
 i.e. $\tilde{\beta}$ is biased, since $\hat{\beta}$ is unbiased.
 Furthermore, the more untrue the restrictions are,
 the more severe is the bias.

Under some circumstances, you may want to
 impose restrictions even though they aren't true.

- This reduces the variance;
- This introduces some bias.

Hence we are considering the tradeoff that
 appears so often; between reducing the variance
 and accepting some bias.

\therefore Considering some estimation criterion such as min. MSE.



Should!

Note: The decision to impose restrictions will hinge on how much the variance is reduced, and how much bias is introduced.

))) i.e. How "wrong" are the restrictions?

More later!

★ The implications of imposing restrictions is analagous to omitting relevant variables: variance reduced, bias introduced. Indeed omitting variables is one way to impose "restrictions."

The Non Centrality Parameter :

$$\lambda = \frac{(R\beta - r)' [R(X'X)^{-1}R']^{-1} (R\beta - r)}{2\sigma^2}$$

This is a measure of how wrong the restrictions are. If the restriction is true, $\lambda = 0$. If the restriction is more wrong, λ is larger.

Fact: $MSE(\tilde{\beta}) \leq MSE(\hat{\beta})$ iff $\lambda \leq \frac{1}{2}$.

See ~~Dudley Wallace, JASA, 1968.~~
Toro-Vizcarrondo & Wallace, JASA, Jr, 1968.

Note - if $\lambda \leq \frac{1}{2}$, the restrictions are "close to true."

★ Useful! This measure tells you if the restrictions hold to the extent that you'll get a "better" estimate by imposing them. Estimate $\hat{\lambda}$ with $\hat{\beta}$!