3. VIOLATIONS OF THE IDEAL CONDITIONS

This ends our discussion of the Classical Regression Model under the ideal conditions.

∴ End of Statistician's part of the course.

Now consider Econometrician's part – violations.

In problems that economists face, in Acctng & Finance ideal conditions often do not hold.

∴ The nice properties of OLS estimates are brought into question.

Not only are the (i) <u>hypothesis tests</u> often <u>invalid</u> (since the distributions of the OLS estimates depend on the ideal conditions),

but there are (ii) <u>other ways to estimate</u> the model that will be <u>better</u> than OLS.

Restrictions

We can sometimes impose restrictions on our model <u>if</u> <u>we</u> <u>know</u> <u>they</u> <u>are</u> <u>true</u>.

This is done to gain efficiency in estimation → to get better estimates.

Consider <u>Exact Linear Restrictions</u>

$$Y = X\beta + \epsilon \qquad \text{with ideal conditions holding}$$

Restrictions: $\underset{(m\times K)}{R}\,\underset{(K\times 1)}{\beta} = \underset{m\times 1}{r}$

Consider the "restricted estimator",

$$\tilde{\beta} = \hat{\beta} + (X'X)^{-1}R'\left[R(X'X)^{-1}R'\right]^{-1}(r - R\hat{\beta})$$

where $\hat{\beta} = (X'X)^{-1}X'Y$ = OLS estimate.

Lagrange Mult.
recall First Order Conditi
Max π s.t. budget constrai
Max utility s.t. income co

<u>Properties of $\tilde{\beta}$:</u>

nder Ho: $R\beta = r$,

1) $\tilde{\beta}$ maximizes the likelihood function (and equivalently minimizes SSE) s.t. the restrictions, $R\beta = r$.
2) $R\tilde{\beta} = r$ [see EXAM] do on bd! (easy)
3) $\tilde{\beta}$ is efficient relative to $\hat{\beta}$; $[Cov(\hat{\beta}) - Cov(\tilde{\beta})]$ is ps

[illegible]

This 3[rd] property is true; $[Cov(\hat{\beta}) - Cov(\tilde{\beta})]$ is psd.; whether or not the imposed restrictions, $R\beta = r$, are true. Hence, imposing the restrictions reduces the variance, though it will introduce <u>bias</u> <u>if</u> the restrictions are <u>not</u> <u>true</u>.

<u>Intuition:</u> By imposing restrictions, you constrain the model; you reduce the number of things in the model that can vary.

$\Rightarrow$ $\therefore$ You reduce the variability of the model.

But if $R\beta \neq r$ in reality, then $\tilde{\beta} = \hat{\beta} + [\text{something with Expected Value} \neq \underline{0}]$.
i.e. $\tilde{\beta}$ is biased, since $\hat{\beta}$ is unbiased.
Furthermore, the more <u>untrue</u> the restrictions are, the more severe is the bias.

Under some circumstances, you <u>may</u> want to impose restrictions even though they <u>aren't</u> true.

— This reduces the variance;
— This introduces <u>some</u> bias.

Hence we are considering the tradeoff that appears so often; between reducing the variance and accepting some bias.

$\therefore$ Considering some estimation criterion such as min. <u>**MSE**</u>.

Should!

Note: The decision to impose restrictions will hinge on how much the variance is reduced, and how much bias is introduced.

i.e. How "wrong" are the restrictions?

More later!

(★) The implications of imposing restrictions is analagous to omitting relevant variables: variance reduced, bias introduced.
Indeed omitting variables is one way to impose "restrictions."

The Non Centrality Parameter:

$$\lambda = \frac{(R\beta - r)'[R(X'X)^{-1}R']^{-1}(R\beta - r)}{2\sigma^2}$$

This is a measure of how wrong the restrictions are.
If the restriction is true, $\lambda = 0$.
If the restriction is more wrong, λ is larger.

Fact: $MSE(\tilde{\beta}) \leq MSE(\hat{\beta})$ iff $\lambda \leq \frac{1}{2}$.

See ~~Dudley Wallace, JASA, 1968~~.
(Toro-Vizcarrondo & Wallace, JASA, Je., 1968.)

Note - if $\lambda \leq \frac{1}{2}$, the restrictions are "close to true."

★ Useful! This measure tells you if the restrictions hold to the extent that you'll get a "better" estimate by imposing them. Estimate $\hat{\lambda}$ with $\hat{\beta}$!