SIMULTANEOUS EQUATIONS

INTRODUCTION

Example 1: Simple Keynesian Model

Structural Equations

$$C = \alpha + \beta Y \qquad \text{endog.} - C, Y$$
$$Y = C + I \qquad \text{exog.} - 1, I$$

What is β? MPC? *Wrong!*

MPC is $\frac{\partial C}{\partial Y}$. This $= \beta$ only if $\nexists$ no further affects (ceterus par.) of cons. on income, then income on cons., then cons. on income,
⋮

The second equation represents a link between C & Y. The MPC is the increment in C, given an ↑ in Y; But the second equation links C & Y! Thus, we cant ↑ Y by one unit without affecting the system.

∴ The MPC is a conceptual question of what would happen to C *if* we changed Y one unit. —(ceteris paribus)
We can't just do that because of the presence of the second equation (simultaneity).

— Y is determined endogenously.

Reduced Form [endog. in terms of exog.]

$$C = \alpha + \beta(C + I)$$

$$Y = (\alpha + \beta Y) + I$$

$$C = \frac{\alpha}{1-\beta} + \frac{\beta}{1-\beta} I$$

$$Y = \frac{\alpha}{1-\beta} + \frac{1}{1-\beta} I$$

This is not yet a statistical system.

To get a statistical system,
we need a disturbance term.

$$C_t = \alpha + \beta y_t + \epsilon_t$$

$$Y_t = C_t + I_t \qquad \leftarrow \text{Identity}$$

Assume ϵ_t is nicely behaved,
and I_t fixed, or at least uncorrel. with ϵ_t.

$$C_t = \frac{\alpha}{1-\beta} + \frac{\beta}{1-\beta} I_t + \frac{1}{1-\beta} \epsilon_t$$

$$Y_t = \frac{\alpha}{1-\beta} + \frac{1}{1-\beta} I_t + \frac{1}{1-\beta} \epsilon_t$$

We <u>can</u> apply OLS to the reduced form equations.

We <u>can't</u> apply OLS to the structural equations;
Y_t is correlated with ϵ_t,
resulting in biased and inconsistent est.'s.

→ A case 3 (Stochastic Regressors) problem.
→ Simultaneous Equations Bias.

In general, the <u>Simultaneous Equations Problem</u> is that structural equations which include endogenous variables on rhs will yield biased & inconsistent OLS estimates.

ReWrite Example 1 in

Useful Notation for Structural Models

$$-1 \cdot C_t + \beta Y_t + \alpha \cdot 1 + 0 \cdot I_t + \epsilon_t = 0$$

$$1 \cdot C_t + -1 \cdot Y_t + 0 \cdot 1 + 1 \cdot I_t + 0 = 0$$

OR: $$\begin{bmatrix} C_t & Y_t \end{bmatrix} \begin{bmatrix} -1 \\ \beta \end{bmatrix} + \begin{bmatrix} 1 & I_t \end{bmatrix} \begin{bmatrix} \alpha \\ 0 \end{bmatrix} + \begin{bmatrix} \epsilon_t \end{bmatrix} = 0$$

$$\begin{bmatrix} C_t & Y_t \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 & I_t \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} = 0$$

OR: $$\underbrace{\begin{bmatrix} C_t & Y_t \end{bmatrix}}_{\text{vector of endog. var.'s}} \begin{bmatrix} -1 & 1 \\ \beta & -1 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & I_t \end{bmatrix}}_{\text{vector of exog. var.'s}} \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \epsilon_t & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

OR: $$Y\Gamma + X\Delta + \epsilon = 0$$

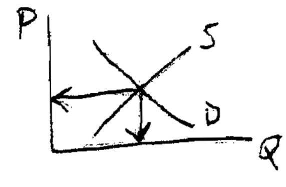
where Y = vector of endogenous variables
Γ = matrix of coefficients on endog. var.'s
X = vector of exogenous variables
Δ = matrix of coefficients on exog. var.'s
ϵ = disturbance terms

Example 2: Supply and Demand Model

$$Q_t = a + b P_t + \epsilon_{t1} \qquad \text{(Demand)}$$

$$Q_t = \alpha + \beta P_t + \gamma W_t + \epsilon_{t2} \qquad \text{(Supply)}$$

Q_t, P_t endogenous
1, W_t exogenous



Rewrite system:

$$[Q_t \;\; P_t] \begin{bmatrix} -1 & -1 \\ b & \beta \end{bmatrix} + [1 \;\; W_t] \begin{bmatrix} a & \alpha \\ 0 & \gamma \end{bmatrix} + [\epsilon_{t1} \;\; \epsilon_{t2}] = 0$$

$$Y \quad \Gamma \quad + \quad X \quad \Delta \quad + \quad \epsilon \quad = 0$$

Example 3:

$$C_t = \alpha + \beta Y_t + \gamma C_{t-1} + \epsilon_t$$

$$Y_t = C_t + I_t$$

endogenous : C_t, Y_t
exogenous : 1, I_t
lagged endog. : C_{t-1} ← [determined by model, but not in time t]

Stochastic Regressors Case 2

If ϵ_t is nicely behaved, then C_{t-1} is uncorrelated with ϵ_t.

→ Lagged endogenous variables often act like exogenous variables.
So group exogenous and lagged endogenous together & call them "Predetermined Variables."

Rewrite System:

$$[C_t \;\; Y_t]\begin{bmatrix} -1 & 1 \\ \beta & -1 \end{bmatrix} + [1 \;\; I_t \;\; C_{t-1}]\begin{bmatrix} \alpha & 0 \\ 0 & 1 \\ \gamma & 0 \end{bmatrix} + [\epsilon_t \;\; 0] = 0$$

$$Y \quad \Gamma \quad + \quad X \quad \Delta \quad + \quad \epsilon \quad = \quad 0$$