

#### Computer Problem 4: Redoing Gini Ratio Study

The residuals of the model for Computer Problem 1 show heteroskedasticity.

$$\text{Gini}_i = a + b_1 (\text{GDP}/N)_i + b_2 (\text{GDP}/N)_i^2 + \varepsilon_i \quad (1)$$

However,  $\text{Var}(\varepsilon)$  is not constant. This variance gets smaller as  $\text{GDP}/N$  increases, across countries.

To fix the heteroskedasticity problem, must make an assumption.

$$\text{Assumption 1: } \text{Var}(\varepsilon_i) = [1/(\text{GDP}/N)_i] \sigma^2. \quad (2)$$

To fix this problem, given this assumption, consider the following GLS-transformation:

$$\text{Let } V = 1/[\text{Var}(\varepsilon_i)] = 1/[1/((\text{GDP}/N)_i)^{1/2}] = \text{GDP}/N_i^{1/2}. \quad (3)$$

Then the GLS-transformed model to analyze using OLS is:

$$VY = VX\beta + V\varepsilon, \quad \text{or:}$$

$$(\text{Gini}_i) * (\text{GDP}/N_i^{1/2}) = a + b_1 (\text{GDP}/N_i^{1/2}) * (\text{GDP}/N_i) + b_2 (\text{GDP}/N_i^{1/2}) * (\text{GDP}/N_i^2) + (\text{GDP}/N_i^{1/2}) * \varepsilon_i \quad (4)$$

Note that, given Assumption 1, this transformed model has a constant variance:

$$\text{Var}((\text{GDP}/N_i^{1/2}) * \varepsilon_i) = (\text{GDP}/N_i) * \text{Var}(\varepsilon_i) = (\text{GDP}/N_i) * [1/(\text{GDP}/N)_i] \sigma^2 = \sigma^2. \quad (5)$$

Run this GLS transformed model in (2) above.

Provide a valid test of the null hypothesis that  $b_2 = 0$ .

Check the residuals of the transformed OLS regression, to investigate whether the residuals of the transformed model have a constant variance.