
THE EFFECT OF THE HEDGE HORIZON ON OPTIMAL HEDGE SIZE AND EFFECTIVENESS WHEN PRICES ARE COINTEGRATED

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This study compares two alternative regression specifications for sizing hedge positions and measuring hedge effectiveness: a simple regression on price changes and an error correction model (ECM). We show that, when the prices of the hedged item and the hedging instrument are cointegrated, both specifications yield similar results which depend on the hedge horizon (i.e., the time frame for measuring price changes). In particular, the estimated hedge ratio and regression R^2 will both be small when price changes are measured over short intervals, but as the hedge horizon is lengthened both measures will converge toward one. These results imply that, when prices are cointegrated, a longer hedge horizon will yield an optimal hedge ratio closer to one, while at the same time enhancing the ability to qualify for hedge accounting. © 2011 Wiley Periodicals, Inc. *Jrl Fut Mark* 32:837–876, 2012

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1. INTRODUCTION

Risk managers conduct statistical analysis to estimate optimal hedge ratios and measure anticipated hedge effectiveness, in order to guide their risk management activities. The standard textbook approach for this task is to regress price changes of the hedged item on price changes of the hedging instrument (e.g., a futures contract), where the price changes are measured over a time span equal to the hedge horizon. The resulting slope coefficient is the optimal hedge size (i.e., hedge ratio), and the R^2 is a measure of hedge effectiveness. Use of regression analysis is reinforced by accounting guidance in the Accounting Standards Codification Topic 815 (formerly Financial Accounting Standard or FAS #133), which requires hedgers to validate their expectation that a prospective derivatives hedge will be effective in offsetting a particular exposure. When regression is used for this purpose, a necessary condition to qualify for special hedge accounting treatment is that the R^2 statistic must be no less than 0.80.

This study compares the textbook approach with an error correction model (ECM), as the hedge horizon is varied. In theory, the proper specification depends on whether the price series have unit roots and are cointegrated. Such a cointegrated relation is expected if the hedged item and the hedging instrument are based on the same underlying asset. We are interested in how these two specifications behave as we extend the hedge horizon, when the price series are cointegrated. It is critical for hedgers to understand how these specifications perform, so they can make informed decisions as they manage risk while complying with hedge accounting rules.

We analyze the theoretical relation between the hedge horizon and the hedge ratio and regression R^2 , when the prices of the hedged item and hedging instrument are based on the same underlying asset, and are cointegrated. For a prototypical ECM, we prove that the estimated hedge ratio and regression R^2 will both be small for short horizons. However, as the hedge horizon is extended, the ECM will generate an error correction coefficient that converges toward negative one, whereas the hedge ratio and R^2 increase toward positive one. Furthermore, the latter two results also occur with the standard textbook approach, when the user omits the error correction term and estimates a simple regression on price changes.

We empirically explore these issues with a case study that analyzes optimal hedge size and hedge effectiveness for a firm using gasoline futures to hedge delivery of gasoline at six locations. We find the daily gasoline cash and futures prices are cointegrated, in line with the theoretical prototypical ECM. Then, consistent with our analytical results, we also find the ECM and the textbook approach both yield a slope coefficient and regression R^2 which are small when we use a daily hedge horizon, but which increase toward one as we extend the

time frame for measuring price changes to weekly, monthly, quarterly, and six-month intervals. In addition, the error correction coefficient in the ECM approaches negative one as the hedge horizon increases.

These analytical and empirical results have profound implications for the use of, and accounting for, derivatives in hedging. First, if the cash and futures prices follow a stable relation consistent with the prototypical ECM, then it is reasonable to anticipate that the optimal hedge will ultimately be effective in offsetting the exposure, provided that the hedger can maintain the positions for a sufficiently long hedge horizon. This result, by itself, should obviate the need for any further statistical analysis to assess hedge effectiveness. Second, if data on the cointegrated price series are available for an adequate sample period, then the hedger can be confident in meeting the criterion to qualify for hedge accounting (i.e., obtaining an R^2 close to one), if the hedger is willing to consider a sufficiently long hedge horizon. Third, it does not matter if the user estimates an ECM or applies the standard textbook approach, as both will yield an estimated hedge ratio and R^2 that approach one as we lengthen the hedge horizon.

This article proceeds as follows. Section 2 provides a literature review and motivation for our study. Section 3 discusses the relevant accounting and econometric issues. Section 4 describes our two alternative regression specifications. Section 5 presents our analytical findings regarding the choice of a hedge horizon, and the implications of cointegration for the regression specifications. Section 6 provides the case study. A final section summarizes and concludes.

2. BACKGROUND AND MOTIVATION

2.1. Hedge Ratios, Hedge Effectiveness, and Hedge Accounting

A large body of research addresses how the hedging activities of risk managers can enhance firm value.¹ Risk managers need to assess the proper sizes and anticipated effectiveness of their hedge positions, in order to make informed decisions. Accounting rules under Topic 815 support this need, by requiring hedgers to validate their expectation that a prospective hedge will be “highly” effective in offsetting a particular risk exposure, in order to qualify for special hedge accounting treatment. It is common to use regression analysis for this purpose.

If a company does not qualify for special hedge accounting, derivative positions would be marked to market through current earnings, while the earnings

¹For examples of recent work, see Allayannis and Weston (2001), Carter, Rogers, and Simkins (2006), Jin and Jorion (2006), and Mackay and Moeller (2007).

impacts of the hedged items would likely be reported in later accounting periods. With hedge accounting, on the other hand, these two results are reported concurrently. Thus, hedge accounting has the appeal of making the economic intent of the hedge transparent in the financial statement. To the extent that this outcome would be viewed as a social good, it would seem that the application of hedge accounting should be encouraged. Current accounting rules, however, impose a variety of impediments to implementing this treatment. Specifically, as a prerequisite for hedge accounting, hedgers must demonstrate a basis for believing that their hedges will be “highly” effective in offsetting the risks being hedged.²

2.2. Statistical Measures of the Optimal Hedge Ratio and Anticipated Hedge Effectiveness

The common metric for validating the expectation that a hedge will be “highly” effective is the R^2 from a regression specification, whereas the regression slope coefficient provides an estimate of the optimal hedge ratio. Furthermore, it is understood that a necessary condition to qualify for hedge accounting treatment is a regression $R^2 \geq 0.80$. The proper design of the regression model, however, is a matter of some dispute.³

The textbook prescription for measuring the optimal hedge ratio and hedge effectiveness is a simple regression on price changes, where the dependent variable relates to the risk exposure (e.g., a cash price) and the independent variable relates to the hedging instrument (e.g., a futures price), whereas the time frame for measuring price changes matches the hedge horizon (Ederington, 1979; Hull, 2008). Another common industry practice is to estimate a simple regression on price levels rather than price changes.⁴ In addition, a growing body of work recommends an ECM to account for cointegration.⁵ Moreover, some researchers have explored modeling volatility and asymmetry in the data. For example, Brooks, Henry, and Persaud (2002) employ a cointegrated model that allows for the conditional variances to be asymmetric functions of shocks to spot and futures prices. They show that the optimal hedge

²For more discussion advocating current hedge accounting guidance, see Hodder, Hopkins, and Wahlen (2006), Kanodia, Mukherji, Sapra, H. and Venugopalan (2000), and Zhang (2009).

³The Financial Accounting Standards Board (FASB) has recently released an exposure draft that, if enacted, would lower the requirement from “highly” effective to “reasonably” effective. It is not clear whether or how this change would affect the current practice of requiring the regression R^2 statistic to be at least 0.80.

⁴Levy and Sarnat (1984) and Marmer (1986) discuss various drawbacks to the textbook approach. Hill and Schneeweis (1981, 1982) and Dale (1981) debate use of a simple regression on price levels versus price changes.

⁵For example, see Benz and Hengelbrock (2009), Chou, Fan, Denis, and Lee (1996), Ghosh (1993a,b), Kenourgios, Samitas, and Drosos (2008), Kroner and Sultan (1993), Lien (1996, 2004), and Lien and Tse (1999).

ratio is sensitive to these asymmetries, and that using the improved hedging methodology results in reduced risk at short horizons.

In this study, we focus on the use of R^2 as a measure of hedge effectiveness, given that a regression $R^2 \geq 0.80$ represents one means for classifying a position as a “highly” effective hedge. However, a number of alternative metrics could also serve this role. For example, Cotter and Hanley (2006) illustrate other measures to assess the performance of futures in hedging seven major stock indices, including value at risk (VaR), lower partial moments, semi-variance, and conditional VaR. They find that the use of different metrics for assessing hedge effectiveness may alter the model chosen for constructing an optimal hedge.

The various statistical efforts necessary to qualify for hedge accounting are somewhat cumbersome and costly to implement. Moreover, the lack of specific accounting guidance or consistency regarding the design of the effectiveness tests may permit some hedgers to qualify for hedge accounting while others do not, even when such hedgers face the same underlying economics. Still other hedging entities may refrain from hedging with derivatives altogether, given the costs of compliance and the possibility of failing to qualify for hedge accounting.⁶

2.3. The Effect of the Hedge Horizon on Optimal Hedge Size and Effectiveness

A substantial body of prior work has shown that regression analysis on price changes taken over short time intervals often leads to a small hedge ratio and a low R^2 . However, as the hedge horizon is lengthened, the estimated hedge ratio and R^2 measure both tend to increase.⁷ Such behavior presents an obvious problem for a risk manager who faces a relatively short hedge horizon. A common explanation for this behavior is that short run noise in the market tends to cancel out over longer horizons, as the true long run relation is revealed. This explanation, however, lacks a robust theoretical foundation and sheds little light on the underlying causes.

To better understand this behavior, two prior studies model the relation between the hedge horizon and the optimal hedge ratio or measure of hedge effectiveness. Howard and D’Antonio (1991) build a model in which the optimal hedge ratio depends on the investment horizon. Their results are driven by

⁶For more dialogue on the drawbacks of current hedge accounting guidance, see Charnes, Berkman, and Koch (2003), DeMarzo and Duffie (1995), Freeman and Wells (2009), Kawaller and Koch (2000), Sapra (2002), and Wong (2000).

⁷See Benet (1992), Chou et al. (1996), Ederington (1979), Geppert (1995), Howard and D’Antonio (1991), Hill and Schneeweis (1981, 1982), Lee, Lin, Tu, and Chen (2009), Lien (2000), Lien and Luo (1993), and Malliaris and Urrutia (1991a,b).

an assumption of autocorrelation in the spot asset return. Geppert (1995) analyzes the theoretical relation between the hedge horizon and both the optimal hedge ratio and hedge effectiveness, using the permanent/transitory representation implied by cointegration. Geppert's model implies that the R^2 converges toward one as the hedge horizon increases, whereas the optimal hedge ratio converges to the ratio of sensitivities of the cash and futures prices, respectively, to the permanent component that drives these prices.

We provide an alternative approach to model the relation between the hedge horizon and the optimal hedge ratio and R^2 , when the underlying price series are cointegrated. We contribute to the dialogue by analyzing a theoretical, prototypical ECM to ascertain the effects of increasing the hedge horizon on the behavior of the model, when the prices are cointegrated. We show that, as the hedge horizon is extended, the error correction coefficient tends toward negative one, whereas the estimated hedge ratio and R^2 both approach positive one. In addition, we prove that the latter two results still hold if the hedger omits the error correction term, and applies the standard textbook approach to estimate a simple regression on price changes.

Our model differs from that of Howard and D'Antonio (1991), by assuming that the cash and futures prices are cointegrated according to a prototypical ECM. Our work offers an alternative approach to the model of Geppert (1995), by assuming that only the cash price adjusts to shocks, while focusing on the behavior of the prototypical ECM itself rather than the permanent/transitory representation implied by the cointegrating system. This approach is valuable as most practitioners who analyze cointegrating relations work in the domain of an ECM rather than the theoretical permanent/transitory representation of the cointegrating system.

Our focus on the nature of the ECM offers a number of insights beyond the analysis of Howard and D'Antonio (1991) and Geppert (1995), by helping us to understand how the specific attributes of the ECM affect the hedge ratio and R^2 measure. In particular, we show that the coefficient of the error correction term plays a key role in the relation between the hedge horizon and both the hedge ratio and R^2 measure. First, when this coefficient is zero the prices are not cointegrated, the ECM simplifies to the textbook specification, and the optimal hedge ratio and R^2 measure are independent of the hedge horizon. Second, when this coefficient is non-zero but close to zero, the optimal hedge ratio and R^2 converge to one slowly as the hedge horizon is increased. Third, when this coefficient is non-zero, the magnitude of this coefficient itself converges to negative one as the hedge horizon is extended. Fourth, when this coefficient is non-zero but the user does not include the error correction term in the ECM, the estimated hedge ratio and R^2 measure will still increase toward one as the hedge horizon is lengthened.

3. OVERVIEW OF HEDGE ACCOUNTING AND ECONOMETRIC ISSUES

3.1. The Mechanics of Hedge Accounting

The use of derivative instruments for hedging is straightforward. For any undesired exposure, find a closely related derivative instrument (e.g., a futures contract on a similar underlying price) and take a position as an overlay to the exposure being hedged. A well-functioning hedge would then be expected to offset the impact of an adverse price move on the exposure. Of course, proper sizing of the hedge position is critical to the success of the effort. However, there is no widespread agreement about the proper methodology for determining the optimal hedge ratio. This lack of consensus is highlighted by the hedge documentation requirements that the FASB has stipulated as prerequisite for applying special hedge accounting treatment. This treatment assures that the earnings recognition from hedging transactions will be paired in the same accounting period as the earnings impact from the hedged item. Without hedge accounting, these two effects would likely be reported in different periods. Given the expectation that the derivative will offset the exposure, pairing these two income effects in the same accounting period would tend to lower volatility for reported earnings, as compared to these same earnings impacts being reported in different periods. All else equal, analysts and investors are believed to assign higher stock valuations to companies with lower earnings volatility. Hence the appeal of hedge accounting is understandable.

As appealing as hedge accounting might be, its use is not an election. Rather, hedgers must meet certain requirements to qualify for hedge accounting. In this regard, the finance and accounting literatures distinguish between the special case of a simple hedge, and the general case of a cross-hedge. In a simple hedge, the *underlying good and location* associated with the exposure are *both* identical to those specified for the hedging derivative, a one-to-one hedge ratio is obvious, no analytics are required, and hedge accounting routinely applies.

In a *cross-hedge*, the exposure and the derivative are not based on the same underlying instrument (price) at the same location, and a one-to-one hedge ratio may not be the proper choice. As a result, cross-hedgers are required to conduct a statistical analysis of the hedging relation between the prices of the hedged item and the hedging derivative. This analysis serves both to determine the optimal hedge ratio and to validate their expectation that the hedge will be “highly” effective in offsetting the fair values or cash flows associated with the hedged item.⁸

⁸FAS #133 is notoriously complex, yet purposely vague about the specific analysis required.

It is important to note that, in a cross-hedge, the hedged item and the hedging instrument may represent different assets that could be calibrated in different units of measurement. In such a case the hedge ratio would not normally be expected to converge to one in the cointegrating relation. Instead, the proper hedge size would converge to the appropriate ratio that reflects the normal physical association between the two assets.

In our theoretical analysis, we ignore this latter issue by assuming that the two assets involved in the cross-hedge represent the same underlying good, but at different locations. Thus, the two assets involved in the cross-hedge of our theoretical analysis are expected to embody a normal one-to-one hedging relation in the long run. This situation also represents the problem analyzed in our case study, which examines the hedging relation between the futures price of gasoline for delivery in New York Harbor, and the cash prices of the same commodity (gasoline) for delivery at six different locations. We now turn to the details of this type of cross-hedge.

3.2. Accounting Rules for a Cross-Hedge

Situations involving a cross-hedge are common in managing commodity price risk, where cash market transactions are typically priced with a “basis” or differential relative to a readily observable industry reference price, such as the nearby (i.e., next-to-expire) futures price. For example, gasoline pricing follows this practice, where the cash price at any location is determined relative to the nearby gasoline futures contract traded at the New York Mercantile exchange, which prescribes delivery in New York Harbor. In this case, the underlying prices for the hedged item and the hedging derivative pertain to the same underlying good, but not at the same location. As a result, the basis does not generally converge to zero at maturity.

Prior to the introduction of FAS 133, most hedgers ignored this basis effect in a cross-hedge. That is, gasoline producers elected to hedge what amounted to the “New York Harbor component” of their gasoline price exposure. As long as the associated basis was relatively stable, and/or was only a minor component of the gasoline price (which is typical), these hedgers were largely comfortable that they were addressing the lion’s share of their gasoline price exposure. In these cases the appropriate hedge size remained trivial, and a one-to-one hedge ratio would apply with no regression analysis needed.

FAS 133 changed this practice by restricting hedge accounting treatment only to hedges that strived to offset the full change in the price of the gasoline purchases or sales—i.e., not just the New York Harbor portion, but the basis effects as well. Given this restriction, it became necessary for cross-hedgers to determine the hedge size empirically, as the slope coefficient of a regression analysis. Moreover, auditors have been instructed to confirm that the sizing of

a hedge position implemented by the hedger is consistent with the regression analysis used by the hedger to assess hedge effectiveness. This latter emphasis on proper hedge sizing is an appropriate consideration, as the R^2 statistic would not be a valid indicator of hedge effectiveness if the size of the hedge implemented was not consistent with the slope of the regression model.

3.3. Alternative Regression Models to Estimate Hedge Ratios and Hedge Effectiveness

In theory, the proper specification for the regression model describing the hedging relation depends on the time series behavior of the prices involved. There are three basic cases:

If the price series do not contain unit roots, then a simple regression on either price levels or price changes may be appropriate.

If the price series contain unit roots, but are not cointegrated, then a simple regression on price levels is generally misspecified due to the possibility of spurious regression. In this case a simple regression on price changes (i.e., the standard textbook approach) is appropriate (Granger & Newbold, 1974).

If the price series contain unit roots and are cointegrated, then the textbook approach is misspecified due to omission of relevant variables. In this case the simple regression on price changes can be appended to include an error correction term. The resulting ECM is well-specified in the presence of cointegration.

This study addresses several questions regarding the proper regression design to analyze the hedging relation between the prices of the hedged item and the hedging instrument. In particular, are the presence of unit roots and a significant, stable cointegrating relation sufficient to justify the expectation that a hedge will be highly effective? In this case, are the results from estimating a simple regression on price changes comparable to those from an ECM? For either specification, are the results sensitive to the time frame for measuring price changes (i.e., the hedge horizon)?

In general, improper specification of the regression model may yield unreliable results, casting doubt on the validity of the slope coefficient as the appropriate hedge ratio or the R^2 as an indicator of hedge effectiveness. We are interested in how the textbook approach and the ECM behave when the hedge horizon is lengthened, under conditions of cointegration.

4. ESTIMATING THE HEDGING RELATION

4.1. The Textbook Solution

The standard textbook approach seeks to minimize uncertainty about changes in the value of the combined hedged position, which includes both the exposure

and the derivatives position. This goal can be accomplished by selecting the hedge ratio (h^*) that minimizes the volatility of changes in the combined hedged portfolio. The solution for h^* is the coefficient generated by a simple regression utilizing price changes, as follows (see Ederington, 1979, and Hull, 2008):

$$\Delta C_t = a_C + b_C \Delta F_t + v_t, \quad (1)$$

where ΔC_t = change in the cash price of the hedged item measured over the hedge horizon, ΔF_t = change in the futures price of the hedging instrument over the hedge horizon, v_t = the regression error term, assumed to be stationary, and independent and identically distributed $N(0, \sigma_v^2)$, a_C and b_C are the coefficients from the regression on price changes.

The R^2 from this analysis reflects the proportion of the total variation in cash price changes that is explained by variation in futures price changes. Thus, to the extent that the prospective future relation between changes in cash and futures prices is expected to be similar to that over the historical period analyzed, we can anticipate that using a hedge ratio of b_C should eliminate a proportion of the total risk exposure that is equivalent to this R^2 measure.

4.2. Problems in Applying the Textbook Solution

The textbook solution prescribes analyzing the relation between changes in cash and futures prices, in (1). In conducting this analysis, one critical question regards the appropriate time interval for measuring price changes. The analysis should ideally be limited to the relevant conditions at the start and end of the planned hedge. Thus, the traditional textbook approach recommends applying the regression analysis to data on historical price changes measured over a time interval equal to the hedge horizon (Hull, 2008; pp 54–58). This choice of time frame for measuring price changes, however, is subject to two significant limitations.

First, it is desirable to have a large number of observations on price changes, ΔC_t and ΔF_t , to achieve the asymptotic properties of regression analysis. In many cases, however, it may not be possible to generate a sample of adequate size, as data on futures prices are frequently unavailable for long horizons. Moreover, it is desirable to limit the analysis to futures prices of the most actively traded contracts, as more liquid contracts are more likely to be efficiently priced. However, futures contracts are often only actively traded toward the end of their lives, while they represent the “nearby” (i.e., next-to-expire) contract. For example, although gasoline contracts are currently listed with 18 consecutive monthly expirations at any time, the more distant expiration months are not traded actively. Thus, any effort to construct an extended data

set of actively traded futures prices typically involves blending futures prices associated with multiple expirations, by selecting the price of the nearby contract at any point in time.

A second disconcerting implication of the textbook approach has to do with the fact that, as time passes, the hedge horizon for a given hedging problem is ever-diminishing. Current accounting guidance requires hedgers to provide updated analyses at least quarterly throughout the life of the hedge, which are both retrospective and prospective, to document that their hedges have performed effectively and are expected to continue be effective over their remaining lives.⁹ This requirement for ongoing prospective analyses suggests that the hedger should implement a series of regression tests that incorporate price changes measured over time intervals of different lengths, corresponding to a hedge horizon that diminishes over the life of a hedge. For instance, the hedger with a nine-month horizon remaining would use data reflecting nine-month price changes, whereas three months later the same hedger (now with six months remaining in the hedge horizon) would use six-month price changes in the updated analysis, and so forth.¹⁰

These issues lead to the empirical question of whether the nature and extent of the hedging relation (i.e., the slope coefficient and regression R^2) are sensitive to the time frame used to measure price changes. If the empirical results vary under these respective designs, then it would seem that the hedge ratio should be adjusted throughout the life of a given hedge. However, practitioners generally act as though the nature of the exposure remains unchanged throughout a hedge's life, as they do not adjust hedge ratios during the original hedge horizon.¹¹

4.3. Cointegration and the ECM

Consider the following situation in which C_t and F_t are assumed to represent the cash and futures prices of the same underlying asset, for delivery at two different locations. We further assume that C_t and F_t are each integrated of order one, and follow a simple cointegrating relation in which the cash price responds to deviations from equilibrium, whereas the futures price does not:

⁹This current requirement is also subject to possible changes. The recent exposure draft proposes that the retrospective assessments may only be required if and when there is a material change from the original expectations relating to the hedged item, as of the start of the designated hedging relation.

¹⁰Originally in Derivative Implementation Issue E7, reporting entities were granted permission to apply the same regression design for prospective and retrospective testing. This permission has precipitated the common practice of using the same regression specification to serve double duty. This practice, however, reflects an ignorance of the statistical issues discussed here, relating to the appropriate choice of the time frame for measuring price changes.

¹¹There are exceptions to this generalization, such as "tailed hedges" and delta hedging of option positions (see Kawaller, 1997, and Hull, 2008).

$$C_t = \beta F_t + \varepsilon_{1t}, \quad (2)$$

$$\Delta F_t = \varepsilon_{2t}, \quad (3)$$

where ε_{1t} and ε_{2t} are each independent and identically distributed with mean zero and finite variances, σ_1^2 and σ_2^2 , respectively. Moreover, let the covariance between ε_{1t} and ε_{2t} be given by σ_{12} , and let $\beta \neq 0$. Given these assumptions, both C_t and F_t have unit roots. However the difference, $(C_t - \beta F_t) = \varepsilon_{1t}$, is stationary, and C_t and F_t are cointegrated.¹²

In this case the simple regression on price changes in (1) is misspecified by omission of the error correction term. This conclusion challenges the validity of the standard textbook approach, when the price series are cointegrated. Furthermore, we suggest that cash and futures prices are likely to be cointegrated in many cross-hedging situations, due to arbitrage activity.¹³

When cash and futures prices are cointegrated as in (2) and (3), it is appropriate to estimate the minimum variance hedge ratio using an ECM. This approach focuses on the relation between price changes, as in the textbook approach, but it appends (1) to include a lagged error correction term, ε_{1t-1} , along with lagged values of both cash and futures price changes:

$$\Delta C_t = \alpha_0 + \alpha_1 \varepsilon_{1t-1} + b_{\text{ECM}} \Delta F_t + \sum_{k=1}^m \gamma_k \Delta F_{t-k} + \sum_{j=1}^n \delta_j \Delta C_{t-j} + e_t, \quad (4)$$

where b_{ECM} is the optimal hedge ratio, and the error correction term is $\varepsilon_{1t-1} = (C_{t-1} - \beta F_{t-1})$. This term can be determined as the lagged residual from (2), by joint estimation of (2), (3), and (4).¹⁴

In addition, a measure of anticipated hedge effectiveness that pertains to the estimated hedge ratio from the ECM (b_{ECM}) can be constructed as follows:

$$R^2 \text{Analogue} = 1 - \text{SSE}^* / \text{SST}^*, \quad (5)$$

where SSE^* = the total variation in the time series, $\{\Delta C_t - b_{\text{ECM}} \Delta F_t\}$, about its mean, and SST^* = the total variation in the time series, $\{\Delta C_t\}$, about its mean.

¹²See Enders (2003; pp 363–377). Note that the error term in this cointegrating relation, $\varepsilon_{1t} = (C_t - \beta F_t)$, is analogous to the value of the short hedger's combined hedged position, when β is used as the hedge ratio.

¹³For discussion and evidence supporting this view, see Benz and Hengelbrock (2009), Chou et al. (1996), Ghosh (1993a,b), Kenourgios et al. (2008), Kroner and Sultan (1993), Lien (1996, 2004), Lien and Luo (1993), and Lien and Tse (1999). In contrast, Bailey and Myers (1991) argue that cash and futures prices for commodities may not be cointegrated.

¹⁴In this model, enough lags are included (i.e., m and n are selected to be sufficiently large) to ensure that: (i) all relevant information about ΔF_{t-k} and ΔC_{t-i} is incorporated, and (ii) the error term from the ECM, e_t , is not autocorrelated. In our analysis, the initial choice for m and n is made using the Akaike Criterion. We then apply the Ljung and Box (1978) test on the autocorrelation function of the ECM residuals, to investigate the null hypothesis that e_t is not autocorrelated. If the lag lengths indicated by the Akaike Criterion do not result in white noise residuals, we increase the lag length on the dependent variable (n) further until the Ljung-Box test does not reject the white noise hypothesis.

This measure of hedge effectiveness is analogous to the R^2 from (1), except that SSE^* omits the intercept in (1) and assumes that the hedger uses b_{ECM} as the hedge ratio, rather than b_C .¹⁵

5. THE HEDGE HORIZON, THE HEDGE RATIO, AND THE R^2 FROM AN ECM

In this section we further examine the simple cointegrating relation in (2)–(4). Specifically, we analyze the effect of increasing the length of the hedge horizon (and thus the time frame for measuring price changes) on the estimated hedge ratio and R^2 from the ECM in (4).

5.1. A Prototypical ECM

Consider the relation between the cash price of a commodity for delivery to location a at time t (denoted C_{at}) and the futures price (F_t) for future delivery of the same commodity to a different location. This situation represents a cross-hedge. We posit a joint determination of cash and futures prices at the frequency of observation. In our case study, the commodity is gasoline and the data are observed at the daily frequency. We define the vector of cash and futures prices as:

$$X_t = \begin{pmatrix} C_{at} \\ F_t \end{pmatrix}.$$

Denote $\Delta X_t = X_t - X_{t-1}$, and consider the following assumptions for the joint data-generating process of the cash and futures price series:

Assumption 1: $\Delta X_t = \Pi X_{t-1} + \varepsilon_t$ where ε_t is a martingale difference.

Assumption 2: Suppose that $\Pi = \alpha\beta^T$ with $\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix}$, $\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, and $E(\varepsilon_t \varepsilon_t^T) = \Omega = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$.

Assumption 3: $-1 < \alpha_1 < 0$.

Assumption 4: ε_t is elliptically symmetric.

¹⁵Note that the R^2 from estimating (1) will necessarily be less than the R^2 from the ECM in (4). However, the R^2 from (1) will be greater than the R^2 analogue in (5), as the R^2 from (1) is the maximum R^2 possible for the simple regression of ΔC_t on ΔF_t .

5.2. How the Assumptions Affect the Behavior of the Prototypical ECM

The first assumption states that an ECM characterizes the cointegrating relation between the cash and futures price series. The second and third assumptions impose a structure for Π , in which: $-1 < \alpha_1 < 0$, $\alpha_2 = 0$, $\beta_1 = 1$, and $\beta_2 = -1$. The fourth assumption ensures that conditional expectations are linear.¹⁶

These restrictions incorporate several important features for this cointegrating system. First consider the restrictions on α_1 and α_2 . The restriction on α_1 is sufficient for the difference of the cash price series (ΔC_{at}) to be stationary, and for the system to be cointegrated.¹⁷ Requiring α_2 to be zero implies that the cash price series (C_{at}) responds to any disequilibrium in the relation between the two series, but the futures price series (F_t) does not. If the two prices are cointegrated in this fashion, then they tend to move together in a one-to-one ratio over time. For example, this would likely be the case if arbitrageurs enforce the law of one price in the relation between the cash and futures prices for the same asset at the same location.

Next consider the restrictions on β_1 and β_2 . Together, these restrictions imply that cash price changes (ΔC_{at}) respond to the lagged value of the cash-to-futures basis, ($C_{at-1} - F_{t-1}$). That is, these restrictions imply that the general error correction term, $\varepsilon_{1t-1} = (\beta_1 C_{at-1} + \beta_2 F_{t-1})$, in the ECM simplifies to the cash-to-futures basis, ($C_{at-1} - F_{t-1}$).

For more intuition regarding the behavior of this prototypical ECM, consider a shock that leads to a deviation between cash and futures prices, ($C_{at} - F_t$), which exceeds the no-arbitrage trading range implied by the cost of carry and transaction costs. As $-1 < \alpha_1 < 0$ and $\alpha_2 = 0$, the cash price will respond to arbitrage activity induced by such deviations and adjust back toward the equilibrium trading range, but the futures price will not. The restriction on α_2 is often justified in practice, as it implies that F_t does not adjust to past information, but rather depends upon anticipated future information. This cointegrating system characterizes a market in which the futures price serves a price discovery function for cash prices. There is ample support for such price discovery in futures markets.¹⁸

¹⁶This assumption of linear conditional expectations is routine in asset pricing theory. For example, Owen and Rabinovich (1983) show that elliptical symmetry of the error structure is sufficient to generate the CAPM, while Berk (1997) proves that this assumption is not only sufficient, but necessary for the CAPM to be linear.

¹⁷See Johansen (1996) Ex. 4.1, for the conditions for cointegration and stationarity of the differences in this model.

¹⁸For example, see Benz and Hengelbrock (2009), Brandt, Kavajecz, and Underwood (2007), Chen and Gau (2009), Kawaller, Koch, and Koch (1987), Quan (1992), Rosenberg and Traub (2006), and Tse, Xiang, and Fung (2006).

In our case study on gasoline futures and cash prices, we provide empirical support for the restrictions implied by the first three assumptions in this prototypical ECM.

5.3. The Hedge Horizon and the Hedge Ratio, the R^2 , and the Error Correction Term

Given the assumption that the data are observed at a daily frequency, the ECM associated with this cointegrated system applies to daily price changes. However, this daily cointegrated system also implies an analogous ECM when longer time frames are used to measure price changes. We define the N -day price differences as $\Delta_N C_{at}(= C_{at} - C_{at-N})$ and $\Delta_N F_t(= F_t - F_{t-N})$. This notation enables us to examine the implications for estimating this ECM when we increase the time frame for measuring price changes (i.e., when we increase the length of the hedge horizon, N).

5.3.1. The Hedge Horizon and the Optimal Hedge Ratio

For a hedge horizon covering N days ahead, the optimal hedge ratio is calculated by estimating an ECM in which the dependent variable is the N -day change in the cash price ($\Delta_N C_{at}$), and the two key right-hand-side variables are: the N -day lag of the basis ($C_{a,t-N} - F_{t-N}$), and the N -day change in the futures price ($\Delta_N F_t$).¹⁹ In general, the optimal hedge ratio for an N -period-ahead hedge is given by the coefficient of $\Delta_N F_t$ in this ECM, which can be characterized as follows:

$$\frac{\text{Cov}(\Delta_N C_{at}, \Delta_N F_t | C_{a,t-N} - F_{t-N})}{\text{Var}(\Delta_N F_t | C_{a,t-N} - F_{t-N})}$$

One goal of this study is to examine the behavior of this optimal hedge ratio as we lengthen the hedge horizon (i.e., as we increase N). To this end, we find an analytical representation of this optimal hedge ratio that is implied by the prototypical ECM, in the following theorem:

Theorem 1: Suppose that Assumptions 1–3 hold. Then the optimal hedge ratio for N periods ahead is given by:

$$\left[\frac{1 - (1 + \alpha_1)^N}{N\alpha_1} \right] \left(1 - \frac{\sigma_{12}}{\sigma_2^2} \right) + 1.$$

All proofs are provided in the Appendix.

¹⁹In addition, it is standard to include lagged values of futures and cash price changes, $\Delta_N F_{t-k}$ and $\Delta_N C_{t-i}$, as in (4). Note that this specification presumes the use of non-overlapping data on the N -day price changes.

Given assumption 3 the term, $(1 + \alpha_1)^N$, converges to zero as N increases, so that the optimal hedge ratio will converge to one as we increase the hedge horizon. Moreover, for $N = 1$, the optimal hedge ratio simplifies to σ_{12}/σ_2^2 , which is the classic textbook solution. Note that this expression does not yield a one-for-one hedge ratio at shorter hedge horizons. To illustrate this point, suppose the following parameter values characterize the cointegrating relation: $\sigma_{12} = 0.5$, $\sigma_2^2 = 1.0$, and $\alpha_1 = -0.1$. Based on these values, the optimal hedge ratio for $N = 1$ -period-ahead implied by the prototypical ECM is 0.5. Then the optimal 2-, 10-, 30-, and 60-period-ahead hedge ratios are 0.525, 0.674, 0.840, and 0.917, respectively.

We also note that, if α_1 is closer to zero, then the optimal hedge ratio converges to one more slowly as we lengthen the hedge horizon. Hence, it is important to estimate the value of α_1 in the ECM, to determine the speed of convergence of the optimal hedge ratio toward one. In particular, many hedgers use a default one-for-one hedge ratio, regardless of the hedge horizon. The value of α_1 determines the potential loss relative to the optimal hedge from such a procedure.

In contrast, if $\alpha_1 = 0$, we have the following corollary:

Corollary 1: Suppose that Assumptions 1 and 2 hold, but $\alpha_1 = 0$. Then, the optimal hedge ratio for N periods ahead is given by:

$$\frac{\sigma_{12}}{\sigma_2^2}.$$

If $\alpha_1 = 0$, then there is no error correction term in the ECM. With no error correction term, there is no adjustment back toward the equilibrium relation following deviations of the basis outside the no-arbitrage trading range, and thus there is no cointegrating relation. As the optimal hedge ratio indicated in this corollary does not depend on the length of the hedge horizon (N), we would expect to find that the estimated hedge ratio does not change as we vary the time frame for measuring price changes (i.e., the hedge horizon), when there is no cointegrating relation.

5.3.2. The Hedge Horizon and the Regression R^2

To measure hedge effectiveness for compliance with FAS 133, hedgers may use the R^2 from a regression of $\Delta_N C_{at}$ on $(C_{at-N} - F_{t-N})$ and $\Delta_N F_t$. The following theorem provides an expression for the population R^2 from this regression, which depends on the parameters of our model:

Theorem 2: Suppose that Assumptions 1–4 hold and $\alpha_1 \neq 0$. Then the population R^2 from a regression of $\Delta_N C_{at}$ on $(C_{at-N} - F_{t-N})$ and $\Delta_N F_t$ is given by:

$$R^2 = 1 - \left(\frac{\left[\frac{\left(\frac{c_1}{N} + \sigma_2^2\right) - \frac{\left(\frac{c_2}{N} + \sigma_2^2\right)^2}{\sigma_2^2}}{\frac{c_3}{N} + \sigma_2^2} \right]}{\right)}$$

where

$$c_1 = \left[\frac{1 - (1 + \alpha_1)^{2N}}{1 - (1 + \alpha_1)^2} \right] (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (2\sigma_2^2 - 2\sigma_{12})$$

$$c_2 = \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12})$$

$$c_3 = c_1 + \left[\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right] [1 - (1 + \alpha_1)^N]^2$$

From this result we see that, as N approaches infinity, the terms, c_1 , c_2 , and c_3 , are bounded so that the R^2 converges to 1. From a practical perspective this result means that, if the prototypical ECM is the appropriate model, then hedgers can be confident that their hedge will be effective and their statistical analysis will support hedge accounting treatment, if they are willing to consider a sufficiently long hedge horizon.²⁰

The following result deals with the case when there is a cointegrating relation, so an error correction term belongs in the ECM, but we fail to include this term in the regression model:

Corollary 2: Suppose that Assumptions 1–4 hold and $\alpha_1 \neq 0$. Then the population R^2 from a simple regression of $\Delta_N C_{at}$ on $\Delta_N F_t$ is given by:

$$R^{*2} = R^2 + \frac{\frac{c_4}{N}}{\frac{c_3}{N} + \sigma_2^2},$$

²⁰Of course, this would require sufficient data on past cash and futures price changes to have ample degrees of freedom to estimate the ECM, as we lengthen the time frame for measuring price changes (i.e., as we increase N).

where R^2 and c_3 are defined as in Theorem 2, and

$$c_4 = [(1 + \alpha_1)^N - 1]^2 \left(\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right).$$

This corollary means that, if $\alpha_1 \neq 0$ so that the error correction term belongs in the model, but we exclude it from the regression, the resulting R^2 will still increase toward one as we lengthen the hedge horizon in the same way that it does when we correctly specify the ECM.

On the other hand, if $\alpha_1 = 0$, then we obtain different results for R^2 when we estimate the ECM with and without the error correction term. This result is shown in the following theorem:

Theorem 3: Suppose that Assumptions 1, 2, and 4 hold, but $\alpha_1 = 0$. Then the population R^2 from a regression of $\Delta_N C_t$ on $(C_{t-N} - F_{t-N})$ and $\Delta_N F_t$ is given by:

$$\frac{\sigma_{12}^2}{\sigma_1^2 \sigma_2^2}.$$

Note that, if $\alpha_1 = 0$, the population R^2 does not depend upon N . Once again, in this situation the price series are not cointegrated and the error correction term does not belong in the regression. Evidently, the adjustment of cash prices to past deviations from equilibrium, via the coefficient α_1 in this cointegrating relation, is the driving force behind Theorems 1 and 2.

5.3.3. The Hedge Horizon and the Coefficient of the Error Correction Term

Corollary 3: The coefficient of the error correction term in the regression of $\Delta_N C_{at}$ on $(C_{at-N} - F_{t-N})$ and $\Delta_N F_t$ is given by $[(1 + \alpha_1)^N - 1]$.

Given assumption 3, the term, $(1 + \alpha_1)^N$, converges to zero as N increases, so the coefficient of the error correction term will theoretically converge to negative one. This result implies that the error correction will become more important in the ECM as we lengthen the hedge horizon.

6. A CASE STUDY

We illustrate the implementation of this analysis by examining proprietary data from a firm that uses gasoline futures to hedge the prospective delivery of gasoline at six locations: Alabama, Chattanooga, Memphis, Nashville East, Nashville West, and Richmond. Sales at each location are hedged with the same futures contract, known as the RBOB, which prescribes delivery of gasoline in

New York harbor. The basis (i.e., the difference between the RBOB futures price and the cash price at each location) is market-determined. This situation thus represents a cross-hedge that requires analytics to validate the expectation that the hedges will be effective.

6.1. Data Considerations

The starting point and first limitation of the analysis relates to collecting the historical data. We use proprietary data from the company in question, on daily gasoline cash prices for each of the six sales locations listed above. The time frame for the available data is limited to the period extending from January, 2006 through April 20, 2008 (i.e., 578 daily observations). Following prior work, we construct a daily futures price series that is composed of the nearby (i.e., next-to-expire) RBOB contract on any given trading day. These futures prices are rolled over to the next contract after each monthly expiration, on the first business day of the month.

6.2. Tests for Unit Roots and Cointegration

We begin our analysis by conducting preliminary tests for unit roots and cointegration. The unit root tests are presented in Table I, and the cointegration tests appear in Table II. We conduct these tests for the price series taken at several different time intervals that correspond to different hedge horizons, including daily, weekly, and monthly. Given our limited sample period, it is not feasible to conduct these asymptotic tests using price series taken at quarterly or six-month periods, as there are fewer than ten such observations. For each time frame examined beyond one day, our price series constitute the last trading day of the period (i.e., week or month).

The results in Table I indicate that the futures price and the six cash price series all contain unit roots, whether measuring price levels at daily, weekly or monthly intervals. That is, regardless of the time frame for measuring price *levels*, we fail to reject the null of a unit root. In contrast, the null is rejected for most cases when price *changes* are measured at daily, weekly, or monthly intervals. This evidence suggests that these price series are integrated of order one.

Results in Table II further show that the RBOB futures price is significantly cointegrated with the cash prices at all six locations, when daily data are employed. However, the evidence for cointegration is weaker when we use weekly data, as only three of the six weekly cash price series reveal significant cointegrating relations at the 0.05 level. Furthermore, monthly data reveal no significant cointegrating relation between any cash price and the futures price, at this level of significance. The diminishing statistical significance of these tests

TABLE I
Unit Root Tests on Futures and Cash Prices for Gasoline

Price:	F	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
<i>Panel A. Daily Price Level (N = 578 days)</i>							
T-stat	-1.56	-1.81	-1.62	-1.52	-1.73	-1.71	-1.61
<i>Panel B. Daily Price Change (N = 577 daily changes)</i>							
ΔPrice:	ΔF	ΔC ₁	ΔC ₂	ΔC ₃	ΔC ₄	ΔC ₅	ΔC ₆
T-stat	-5.17***	-4.03***	-3.99***	-4.45***	-4.52***	-4.44***	-4.27***
<i>Panel C. Weekly Price Level (N = 120 weeks)</i>							
Price:	F	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
T-stat	-1.35	-1.74	-1.91	-1.84	-1.83	-1.92	-1.65
<i>Panel D. Weekly Price Change (N = 119 weekly changes)</i>							
ΔPrice:	ΔF	ΔC ₁	ΔC ₂	ΔC ₃	ΔC ₄	ΔC ₅	ΔC ₆
T-stat	-3.28*	-4.13***	-4.18***	-4.29***	-3.02	-3.28*	-2.74
<i>Panel E. Monthly Price Level (N = 27 months)</i>							
Price:	F	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
T-stat	-1.10	-1.01	-1.02	-0.99	-1.00	-0.80	-0.91
<i>Panel F. Monthly Price Change (N = 26 monthly changes)</i>							
ΔPrice:	ΔF	ΔC ₁	ΔC ₂	ΔC ₃	ΔC ₄	ΔC ₅	ΔC ₆
T-stat	-2.93	-3.97***	-3.99***	-3.88**	-3.88**	-3.57**	-3.78**

*indicates statistical significance at the 0.10 level; ** at the 0.05 level; and *** at the 0.01 level.

In the Panels above, we present unit root tests for gasoline futures and cash prices over the period, January 1, 2006 through April 20, 2008. The unit root tests are conducted on each spot price, as well as on the futures price. The null hypothesis is that there is a unit root. The test is based on the Elliott, Rothenberg, and Stock (1996) version of the Augmented Dickey Fuller test. For the spot prices, the test is based on the model:

$$\Delta C_{at} = \gamma_0 + \gamma_1 t + (\rho - 1)C_{at-1} + \beta_1 \Delta C_{at-1} + \beta_2 \Delta C_{at-2} + \dots + \beta_k \Delta C_{at-k} + \varepsilon_t,$$

where k is the number of lagged differenced terms which is determined by the Modified Akaike Criterion of Ng and Perron (2001). We give the unit root tests for the futures price and for the cash prices at six locations. The locations are:

$a = 1$ (Alabama), 2 (Chattanooga), 3 (Memphis), 4 (Nashville East), 5 (Nashville West), and 6 (Richmond).

The 1, 5, and 10% critical values for the test are -3.97, -3.41, and -3.12, respectively. A t -statistic smaller than this number implies a failure to reject the null hypothesis of a unit root. Note that we only provide results of the unit root tests for the data measured at daily, weekly, and monthly frequencies. The analogous tests are not conducted for longer time intervals, as there are only nine quarterly observations and only four six-month intervals during our sample period.

for a cointegrating relation at longer time intervals is consistent with a decline in the power of these asymptotic tests, when fewer observations are available.

6.3. The Daily ECM

In Table III we provide the results from estimating the daily prototypical ECM. This daily ECM excludes the contemporaneous change in the futures price that appears in Equation (4). This ECM specification is the standard ECM

TABLE II
Cointegration Tests on Futures and Cash Prices for Gasoline

Cash Price:	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆
<i>Panel A. Daily Price Level (N = 578 days)</i>						
Trace Test	38.35***	49.08***	78.25***	34.34***	34.55***	77.30***
P-value	0.0009	0.0000	0.0000	0.0034	0.0032	0.0000
<i>Panel B. Weekly Price Level (N = 120 weeks)</i>						
Trace Test	27.45**	23.07	28.09**	19.39	19.72	36.91***
P-value	0.0316	0.1074	0.0261	0.2584	0.2402	0.0014
<i>Panel C. Monthly Price Level (N = 27 months)</i>						
Trace Test	19.57	19.30	19.47	19.68	20.53	18.73
P-value	0.2479	0.2635	0.2542	0.2425	0.2005	0.2971

* indicates statistical significance at the 0.10 level; ** at the 0.05 level; and *** at the 0.01 level.

In the Panels above, we provide Johansen’s (1996) test for cointegration based on the bivariate Vector Autoregression given by:

$$\Delta X_t = \mu + \gamma t + \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \dots + \Gamma_k \Delta X_{t-k} + \varepsilon_t$$

Where $X_t = (C_{at}, F_t)'$, the vector containing the gasoline cash and futures price at time t . The rank of Π determines the number of cointegrating vectors. The number of lags, k , is determined by the multivariate Akaike Criterion. The null hypothesis is that no cointegrating vectors exist. Rejection of the null implies cointegration. The 5% critical value of the Johansen’s Trace Statistic is 25.87. We give the Trace Statistic and its associated P -values for this cointegration test for the relation between gasoline futures and cash prices at six locations.

The locations are: $a = 1$ (Alabama), 2 (Chattanooga), 3 (Memphis), 4 (Nashville East), 5 (Nashville West), and 6 (Richmond).

We only provide results of the cointegration tests for the data measured at daily, weekly, and monthly frequencies. The analogous tests are not conducted for longer time intervals, as there are only nine quarterly observations, and four six-month intervals during our sample period.

estimated when the researcher documents a typical cointegrating relation. If the researcher is further interested in estimating the optimal hedge ratio, then the prototypical ECM specification (applied in Table III) is appended to include the contemporaneous change in the futures price (ΔF_t), as in Equation (4). The coefficient of this term (ΔF_t) is the optimal hedge ratio. The appended ECM in (4) then offers an alternative specification to the simple regression in (1), for hedgers wishing to estimate optimal hedge ratios and measure anticipated hedge effectiveness. In our subsequent analysis of the ECM in Table V, we follow this procedure by estimating the appended ECM specified in Equation (4), to estimate the optimal hedge ratio and R^2 measure over longer hedge horizons.

In Table III, we examine whether the empirical results for the daily ECM support the restrictions imposed in our prototypical ECM. Specifically, we test the hypotheses that: (i) $\beta_2 = -1$, (ii) $-1 < \alpha_1 < 0$, and (iii) $\alpha_2 = 0$. Consider each restriction, in turn.

TABLE III
 Estimated Daily Cointegrating Relation and Error Correction Model

<i>Dependent Variable:</i>	C_1	C_2	C_3	C_4	C_5	C_6
<i>Panel A. Cointegrating Relation for Daily Level of Cash Price, C_{at}, from Equation (2)</i>						
Intercept	-0.59	-0.58	-0.53	-0.59	-0.60	-0.48
$C_{at-1} (\beta_1)$	1.0	1.0	1.0	1.0	1.0	1.0
$F_{t-1} (\beta_2)$	-0.982	-0.986	-0.996	-0.992	-0.994	-1.025
std error(β_2)	(0.039)	(0.035)	(0.030)	(0.047)	(0.049)	(0.025)
$T(H_0: \beta_2 = -1)$	-0.46	-0.40	-0.13	-0.17	-0.12	0.98
<i>Panel B. Error Correction Model for Daily Change in Cash Price, ΔC_{at}, analogous to (4)</i>						
Dependent Variable:	ΔC_1	ΔC_2	ΔC_3	ΔC_4	ΔC_5	ΔC_6
Intercept	0.0007	0.0008	0.0008	0.0006	0.0006	0.0009
$\varepsilon_{1t-1} (\alpha_1)$	-0.042	-0.050	-0.049	-0.031	-0.028	-0.065
std error(α_1)	(0.007)	(0.008)	(0.006)	(0.006)	(0.005)	(0.009)
$T(H_0: \alpha_1 = 0)$	-5.82***	-6.24***	-7.61***	-5.44***	-5.69***	-7.35***
$[m, n]^a$	[5,5]	[5,5]	[3,3]	[5,5]	[4,4]	[4,4]
$[L_1, L_2]^b$	[3,2]	[2,2]	[1,2]	[2,4]	[2,3]	[2,2]
Adjusted R^2	0.49	0.45	0.58	0.54	0.52	0.58
Overall F -test	50.4***	43.8***	116.0***	63.1***	70.1***	87.7***
<i>Panel C. Error Correction Model for Daily Change in Futures Price, ΔF_t, analogous to (4)</i>						
Dependent Variable:	ΔF	ΔF	ΔF	ΔF	ΔF	ΔF
Intercept	0.0022	0.0020	0.0021	0.0021	0.0021	0.0021
$\varepsilon_{1t-1} (\alpha_2)$	-0.022	-0.006	-0.023	-0.016	-0.017	-0.014
std error(α_2)	(0.027)	(0.028)	(0.026)	(0.024)	(0.022)	(0.033)
$T(H_0: \alpha_2 = 0)$	-0.81	-0.21	-0.90	-0.68	-0.78	-0.42
$[m, n]^a$	[5,5]	[5,5]	[3,3]	[5,5]	[4,4]	[4,4]
$[L_1, L_2]^b$	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]
Adjusted R^2	0.02	-0.01	0.00	-0.01	-0.01	-0.01
Overall F -test	0.89	0.46	0.79	0.58	0.66	0.56
<i>Panel D. Determinant of the Covariance Structure Between ε_{1t} and ε_{2t}, and Log Likelihood Test</i>						
$ \Omega $	4.0E-07	4.5E-07	3.4E-07	3.2E-07	2.8E-07	3.8E-07
log likelihood	2605.4***	2570.6***	2657.6***	2669.2***	2705.4***	2619.0***

*indicates statistical significance for each T -statistic at the 0.10 level; ** at the 0.05 level; and *** at the 0.01 level.

^aThis row provides lag lengths for the ECM (m and n), indicated by the Akaike Criterion and the Ljung-Box test.

^bThis row presents the number of *significant* lagged coefficients on ΔC_{at-j} (L_1), and the number of *significant* lagged coefficients on ΔF_{t-k} (L_2), in the ECM at the 0.05 level.

First examine the results for the cointegrating equation, (2), in Panel A of Table III. The estimate of β_2 is not significantly different from -1 , for all six models in Panel A. This outcome indicates that the error correction term, $\varepsilon_{1t-1} = (\beta_1 C_{at-1} + \beta_2 F_{t-1})$, can be replaced with the cash-to-futures basis, $(C_{at-1} - F_{t-1})$, in each ECM estimated. We follow this procedure below in Table V, when we estimate the analogous ECM in Equation (4) over longer hedge horizons.

Second, consider the ECM determining ΔC_{at} , in Panel B of Table III. These results indicate a rejection of the null hypothesis that $\alpha_1 = 0$, for the

ECMs involving all six cash price series. This evidence shows that the error correction term is an important determinant of cash price changes, in every daily cointegrating relation analyzed. This result implies that these six cash prices respond to past shocks in the equilibrium arbitrage relation between the futures price and each cash price. In addition the estimated coefficient (α_1) is always between zero and negative one, and is close to zero, consistent with the assumptions of our prototypical ECM. This relatively small estimated value for α_1 implies that the optimal hedge ratio and regression R^2 should both converge slowly toward one, as we increase the number of days in the hedge horizon (i.e., as we increase N).

Third, consider the analogous ECM determining ΔF_t in Panel C of Table III. For all six models, we fail to reject the hypothesis that $\alpha_2 = 0$. This evidence contrasts with that in Panel B, indicating that the error correction term is not an important determinant of futures price changes, in any daily relation analyzed. This result implies that the futures price does not respond to past shocks in the equilibrium relation between the futures price and each cash price. In fact, the individual T -statistics and the overall F -tests in Panel C indicate that there are no significant coefficients at all, in any model for ΔF_t . This evidence supports the specification in (3), in which ΔF_t follows a simple white noise process. In contrast, the evidence in Panel B regarding α_1 discussed above suggests that ΔC_{at} does adjust to such shocks. Together, this evidence indicates that the futures price serves a price discovery function in this cointegrating relation.

Fourth, consider the lagged values of ΔF_{t-k} and ΔC_{at-i} in each of the six estimated ECM's for ΔC_{at} and for ΔF_t , respectively, in Panels B and C of Table III. In every ECM determining ΔC_{at} , there are several lagged coefficients on both ΔF_{t-k} and ΔC_{at-i} that are significant at the 0.05 level. On the other hand, there are no significant lagged values of ΔF_{t-k} or ΔC_{at-i} in any of the six daily ECM's that determine ΔF_{t-k} . This result provides more evidence that each cash price adjusts slowly to shocks in this cointegrating system, whereas the futures price is forward-looking.

Finally, at the bottom of Table III we provide the determinant of the estimated error structure ($|\Omega|$) and the log likelihood test, which indicate further support that a significant daily cointegrating relation exists for all six cash price series.

6.4. The Two Regression Specifications to Size Hedges and Measure Hedge Effectiveness

The empirical evidence in Table III indicates that our prototypical ECM characterizes the daily cointegrating relation between the gasoline futures price and the cash price of gasoline at all six locations. Next, we shed light on the empirical

TABLE IV

Measuring the Nature and Extent of the Hedging Relation Between Changes in Futures and Cash Prices Over Different Time Frames: Simple Regression Model

	$\Delta_I C_1$	$\Delta_I C_2$	$\Delta_I C_3$	$\Delta_I C_4$	$\Delta_I C_5$	$\Delta_I C_6$
<i>Panel A. Daily Change: (N = 577 daily changes)</i>						
b_C	0.03	0.02	0.01	0.01	0.01	-0.004
T-stat	(1.7)	(1.3)	(0.5)	(0.5)	(1.0)	(- 0.2)
R^2	0.005	0.003	0.000	0.001	0.002	0.000
<i>Panel B. Weekly Change: (between closing prices on last day each week; N = 119)</i>						
b_C	$\Delta_5 C_1$ 0.26	$\Delta_5 C_2$ 0.26	$\Delta_5 C_3$ 0.27	$\Delta_5 C_4$ 0.24	$\Delta_5 C_5$ 0.23	$\Delta_5 C_6$ 0.30
T-stat	(4.4)***	(4.3)***	(4.2)***	(4.1)***	(4.1)***	(4.6)***
R^2	0.14	0.13	0.13	0.13	0.13	0.15
<i>Panel C. Monthly Change: (between closing prices on last day each month; N = 26)</i>						
b_C	$\Delta_{21} C_1$ 0.62	$\Delta_{21} C_2$ 0.64	$\Delta_{21} C_3$ 0.66	$\Delta_{21} C_4$ 0.58	$\Delta_{21} C_5$ 0.55	$\Delta_{21} C_6$ 0.71
T-stat	(4.7)***	(5.1)***	(5.1)***	(4.2)***	(4.3)***	(5.3)***
R^2	0.48	0.52	0.52	0.43	0.44	0.54
<i>Panel D. Quarterly Change: (between closing prices on last day each week; N = 8)</i>						
b_C	$\Delta_{63} C_1$ 0.67	$\Delta_{63} C_2$ 0.69	$\Delta_{63} C_3$ 0.70	$\Delta_{63} C_4$ 0.61	$\Delta_{63} C_5$ 0.58	$\Delta_{63} C_6$ 0.83
T-stat	(7.3)***	(6.9)***	(8.7)***	(6.2)***	(4.9)***	(8.1)***
R^2	0.90	0.89	0.93	0.86	0.80	0.92
<i>Panel E. Six-Month Change: (between closing prices on last day every 6-months; N = 3)</i>						
b_C	$\Delta_{121} C_1$ 0.70	$\Delta_{121} C_2$ 0.70	$\Delta_{121} C_3$ 0.75	$\Delta_{121} C_4$ 0.74	$\Delta_{121} C_5$ 0.74	$\Delta_{121} C_6$ 0.79
T-stat	(14.2)***	(11.4)***	(514.6)***	(7.4)***	(5.0)***	(13.9)***
R^2	0.995	0.992	1.00	0.982	0.961	0.995

* indicates statistical significance at the 0.10 level; ** at the 0.05 level; and *** at the 0.01 level.

In the different Panels above, we analyze data on changes in spot and futures prices for gasoline over the period, January 1, 2006 through April 20, 2008, using non-overlapping data on price changes taken over daily, weekly, monthly, quarterly, and six-month periods. We summarize the relevant results from the following simple regression model:

$$\text{Hedging Relation Price Changes } \Delta_N C_{at} = a_C + b_C \Delta_N F_t + e_t, \quad (1)$$

where $\Delta_N C_{at} = C_{at} - C_{at-N}$ = change in cash price over N-day interval; $\Delta_N F_t = F_t - F_{t-N}$ = change in futures price over N-day interval;

with N = approximate number of days in period of difference; weekly ($N = 5$), monthly ($N = 21$), quarterly ($N = 63$), 6-months ($N = 126$); and location $a = 1$ (Alabama), 2 (Chattanooga), 3 (Memphis), 4 (Nashville East), 5 (Nashville West), and 6 (Richmond).

validity of the analytical implications of our prototypical ECM, by estimating the two regression specifications in Equations (1) and (4), respectively, as we vary the hedge horizon and thus the time frame for measuring price changes.

6.4.1. Simple Regression on Price Changes

Table IV presents the results from applying the standard textbook approach, to estimate a simple regression on price changes as specified in (1). Note that, as

we move down successive Panels in Table IV, the time frame for sampling price changes is lengthened and the available sample size diminishes. Consistent with our theoretical analysis of the prototypical ECM in Section 4.3 above, these regression results are sensitive to the time frame for measuring price changes.

For example, when daily price changes are analyzed in Panel A of Table IV, we find no evidence of a significant hedging relation for cash price changes at any location, as the estimated hedge ratio (b_c) and R^2 measure are indistinguishable from zero. However, as we proceed down the successive Panels in Table IV to analyze price changes taken over longer time frames, the magnitude of the estimated hedge ratio and R^2 measure both systematically increase toward one. In fact, when quarterly or six-month intervals are used, the R^2 measures are consistently well above 0.80. This outcome suggests an acceptable level of anticipated hedge effectiveness for these longer hedge horizons, supporting compliance with FAS 133. On the other hand, these sample sizes are too small to offer much confidence, when analyzing price changes measured over quarterly or six-month intervals.

This lack of stability in the estimated hedge ratio and R^2 measure, for analyses based on different time frames, is a concern for proponents of the textbook solution. These results suggest that there is no universally appropriate hedge ratio over the entire life of a gasoline hedge, but rather the hedge size should be adjusted as the remaining hedge horizon varies over time. A strict interpretation of these results would suggest that the hedge ratio should be reduced as time passes and the end of the hedge horizon approaches. In fact, hedging should be terminated well before this end date arrives, as the $R^2 < 0.80$ for hedge horizons less than one quarter.

We do not adhere to such a strict interpretation of these results. Rather, we simply observe that the evidence in Table IV supports the analytical results established in Section 4.3 above. Given that the daily data on gasoline cash and futures prices conform to our prototypical cointegrating relation, Table IV reveals that the standard textbook approach results in an estimated hedge ratio and R^2 measure that are both substantially less than one when price changes are taken over short time intervals, whereas both converge toward one as the hedge horizon is lengthened.²¹

6.4.2. Error Correction Model

Panels A–D of Table V provide the relevant results from estimating the ECM in model (4), as we vary the hedge horizon from daily to weekly, monthly, and

²¹These results are also consistent with prior evidence in other contexts (Ederington, 1979, Geppert, 1995, Howard and D'Antonio, 1991, Hill and Schneeweis, 1982, Kroner and Sultan, 1993, Lee et al., 2009, and Lien, 2000).

TABLE V

Measuring the Nature and Extent of the Hedging Relation Between Changes in Futures and Cash Prices Over Different Time Frames: Error Correction Model

	$\Delta_1 C_1$	$\Delta_1 C_2$	$\Delta_1 C_3$	$\Delta_1 C_4$	$\Delta_1 C_5$	$\Delta_1 C_6$
<i>Panel A. Daily Price Change (N = 577)^a</i>						
α_1	-0.052	-0.066	-0.057	-0.044	-0.030	-0.087
t-stat	(- 8.3)***	(- 10.2)***	(- 10.4)***	(- 9.0)***	(- 6.3)***	(- 12.4)***
b_{ECM}	0.030	0.023	0.013	0.005	0.011	-0.001
t-stat	(2.6)**	(1.9)*	(1.3)	(0.5)	(1.2)	(- 0.1)
R^2 analogue	0.0048	0.0031	0.00019	0.00039	0.0015	0.000025
(m, n, DF)	(2, 2, 568)	(1, 4, 565)	(1, 1, 571)	(1, 4, 565)	(3, 3, 565)	(1, 4, 565)
Ljung-Box χ^2_{24}	26.7	30.9	32.2	22.8	18.3	29.5
(P-value)	(0.32)	(0.16)	(0.12)	(0.53)	(0.79)	(0.20)
<i>Panel B. Weekly Price Change (N = 119)</i>						
	$\Delta_5 C_1$	$\Delta_5 C_2$	$\Delta_5 C_3$	$\Delta_5 C_4$	$\Delta_5 C_5$	$\Delta_5 C_6$
α_1	-0.25	-0.29	-0.30	-0.24	-0.19	-0.47
t-stat	(- 5.4)***	(- 5.9)***	(- 5.8)***	(- 5.5)***	(- 5.1)***	(- 7.3)***
b_{ECM}	0.24	0.23	0.23	0.21	0.20	0.26
t-stat	(6.3)***	(6.0)***	(5.7)***	(5.3)***	(5.6)***	(6.5)***
R^2 analogue	0.141	0.134	0.129	0.125	0.125	0.151
(m, n, DF)	(1, 1, 113)	(1, 1, 113)	(1, 1, 113)	(1, 1, 113)	(1, 1, 113)	(1, 3, 109)
Ljung-Box χ^2_{24}	24.7	20.4	28.3	23.0	27.1	30.1
(P-value)	(0.42)	(0.68)	(0.25)	(0.52)	(0.30)	(0.18)
<i>Panel C. Monthly Price Change (N = 26)</i>						
	$\Delta_{21} C_1$	$\Delta_{21} C_2$	$\Delta_{21} C_3$	$\Delta_{21} C_4$	$\Delta_{21} C_5$	$\Delta_{21} C_6$
α_1	-0.92	-0.91	-1.21	-1.14	-1.11	-1.15
t-stat	(- 3.4)***	(- 3.3)***	(- 4.4)***	(- 4.2)***	(- 4.8)***	(- 3.5)***
b_{ECM}	0.61	0.65	0.59	0.50	0.49	0.73
t-stat	(8.2)***	(9.0)***	(8.0)***	(6.3)***	(7.0)***	(10.8)***
R^2 analogue	0.490	0.528	0.528	0.434	0.444	0.555
(m, n, DF)	(1, 1, 21)	(1, 1, 21)	(2, 1, 19)	(2, 1, 19)	(2, 1, 19)	(1, 1, 21)
Ljung-Box χ^2_6	5.2	3.1	2.5	3.3	2.8	0.6
(P-value)	(0.52)	(0.80)	(0.87)	(0.78)	(0.84)	(0.996)
<i>Panel D. Quarterly Price Change (N = 8)^b</i>						
	$\Delta_{63} C_1$	$\Delta_{63} C_2$	$\Delta_{63} C_3$	$\Delta_{63} C_4$	$\Delta_{63} C_5$	$\Delta_{63} C_6$
α_1	-2.18	-2.27	-2.10	-1.71	-1.57	-2.34
t-stat	(- 2.5)**	(- 2.3)**	(- 3.0)***	(- 2.7)**	(- 2.2)**	(- 1.5)
b_{ECM}	1.01	1.04	0.98	0.88	0.84	1.08
t-stat	(6.5)***	(6.5)***	(9.7)***	(8.1)***	(6.2)***	(6.6)***
R^2 analogue	0.654	0.650	0.780	0.712	0.647	0.843
(m, n, DF)	(1, 1, 3)	(1, 1, 3)	(1, 1, 3)	(1, 1, 3)	(1, 1, 3)	(1, 1, 3)
Ljung-Box χ^2	-	-	-	-	-	-
(P-value)	-	-	-	-	-	-

* indicates statistical significance at the 0.10 level; ** at the 0.05 level; and *** at the 0.01 level.

^a Note that the results of estimating the ECM on daily changes in Panel A of Table V differ slightly from the analogous results presented in Table III. These differences are a result of: (i) replacing the error correction term, ε_{1t-N_t} , with the futures-to-cash basis ($C_{at-N} - F_{t-N}$), and (ii) including the contemporaneous change in the futures price (ΔF_t) in the ECM for Table V.

(Continued)

TABLE V (Continued)

^bNote that “–” indicates insufficient data are available to conduct the Ljung-Box test in Panel D. Analogous tests are not conducted for six-month price changes, as there are only three non-overlapping six-month intervals during our sample period, and thus insufficient degrees of freedom to estimate the ECM.

In the Panels above, we analyze data on changes in spot and futures prices for gasoline over the period, January 1, 2006 through April 20, 2008, using the following Error Correction Model:

$$\Delta_k C_{at} = \alpha_0 + \alpha_1 \varepsilon_{1t-N} + b_{ECM} \Delta_N F_t + \sum_{i=1}^m \gamma_i \Delta_N F_{t-i} + \sum_{k=1}^n \delta_k \Delta_N C_{at-k} + e_t \quad (4)$$

where $\Delta_N C_{at} = C_{at} - C_{at-N}$ = change in spot price at location a (C_{at}) over time interval k ; $\Delta_N F_t = F_t - F_{t-N}$ = change in futures price (F_t) over time interval k ; $\varepsilon_{1t-N} = C_{at-N} - F_{t-N}$ = lagged value of cash-to-futures basis; with N = approximate number of days in period of difference; weekly ($N = 5$), monthly ($N = 21$), and quarterly ($N = 63$); and location $a = 1$ (Alabama), 2 (Chattanooga), 3 (Memphis), 4 (Nashville East), 5 (Nashville West), and 6 (Richmond).

The number of lags, m and n , are determined by the Akaike Criterion and the Ljung-Box test. DF is the degrees of freedom available to estimate the ECM. The T-statistics are constructed using OLS standard errors, as all Ljung-Box tests fail to reject the null hypothesis that the errors are white noise. Price changes are taken between the last days of successive periods.

quarterly time frames. Insufficient data are available to estimate the ECM using six-month price changes.

First note that the daily ECM results in Panel A of Table V are similar to those in Panel B of Table III. The minor differences in Panel A of Table V are due to two changes in the specification of the ECM estimated in Table V, relative to that estimated in Panel B of Table III. In Table V we include ΔF_t in the ECM in (4), and we replace the error correction term, ε_{1t-1} , with the cash-to-futures basis, $(C_{at-1} - F_{t-1})$.²²

Second, consider how the coefficient of the error correction term (α_1) varies as we increase the hedge horizon across the Panels in Table V. As in Panel B of Table III, Panel A of Table V reveals that this coefficient is significantly less than zero but small in magnitude, for daily price changes. As we move down successive Panels in Table V, to consider longer hedge horizons, this coefficient increases in magnitude toward negative one. In fact, for the monthly and quarterly time intervals in Panels C and D, this coefficient often exceeds negative one in magnitude, although it is never significantly different from negative one. This evidence supports the analytical result in Corollary 3, established in Section 4.3 above.

Next we compare the estimated hedge ratios (b_{ECM}) and R^2 measures in Table V with the analogous hedge ratios (b_C) and R^2 measures from the simple regression on price changes from Table IV, as we lengthen the hedge horizon. The results from Panels A–C of Table V are similar to those provided in Panels A–C of Table IV. This evidence indicates that the optimal hedge ratio and R^2 measure are robust when we estimate a simple regression on price changes as

²²This replacement is supported by the result in Panel A of Table III, indicating a failure to reject $H_0: \beta_2 = -1$.

in (1), versus the ECM in (4), to analyze price changes measured at daily, weekly, or monthly intervals.

Panel D of Table V reveals somewhat divergent results for the ECM in (4), versus the simple regression in (1) from Panel D of Table IV, when price changes are measured over quarterly intervals. We now find that the estimated hedge ratio (b_{ECM}) from (4) is close to 1.0, which is somewhat larger than the respective hedge ratio (b_C) from (1), which ranges around 0.70. In addition, now the R^2 analogue is somewhat lower for the quarterly ECM in Table V. Of course, the ECM estimated in Panel D relies on only eight quarterly observations.

The empirical evidence in Table V provides further corroboration for the analytical implications of our prototypical ECM, established in Section 5.3. Given the robust daily cointegrating relation we find between gasoline cash and futures prices, we also find several empirical regularities when we extend the hedge horizon. First, the coefficient of the error correction term is significantly negative but small in magnitude for daily intervals, but it grows in magnitude for longer hedge horizons until it is not significantly different from negative one. Second, the estimated hedge ratio and R^2 measures both increase toward one as we lengthen the time interval for measuring price changes. This evidence indicates that, given a cointegrating relation, the hedger can be confident that the hedge will be effective and the statistical analysis will yield an R^2 that exceeds the threshold of 0.80, if enough data are available and the hedger is willing to consider a sufficiently long hedge horizon. Third, for all hedge horizons analyzed, the hedge ratio and regression R^2 are virtually identical when we estimate an ECM, and when we omit the error correction term and estimate a simple regression on price changes. This outcome indicates that, if the price series are cointegrated, then it does not matter whether the hedger estimates the ECM in (4), or applies the traditional textbook approach in (1).²³

Finally, we emphasize that the empirical evidence in this study relies on fewer observations when longer intervals are used to measure price changes. As the available sample for our case study covers just 27 months, we have non-overlapping data on price changes for just 26 monthly, 8 quarterly, or 3 six-month periods. These small sample sizes make us cautious about generalizing our empirical results, especially for quarterly and six-month horizons. Future

²³In an extension of this case study, we have estimated a simple regression on price levels (C_{at} and F_t), rather than price changes (ΔC_{at} and ΔF_t) as specified in Equations (2) and (4). In contrast to our results in Tables IV and V, we find that the slope coefficient and R^2 measure from the simple regression on price levels are both close to one and remain stable when we vary the time frame for sampling price levels across daily, weekly, monthly, quarterly, and six-month intervals. We emphasize that a simple regression on price levels is well-specified when the price series have unit roots and are cointegrated. This fact can be seen in Equation (2) from our prototypical ECM. Future research should examine the circumstances in which further analysis of price levels can provide relevant information regarding the nature and extent of the hedging relation.

research should consider the minimum degrees of freedom that ought to be required in such analysis, before hedgers and regulators rely upon such results. This paucity of data at long horizons represents the major limitation for a hedger conducting this analysis.

7. SUMMARY AND CONCLUSIONS

This study investigates the relation between the hedge horizon and the estimated hedge ratio and regression R^2 , when the price series involved are cointegrated. This analysis contributes to the dialogue on the proper statistical methodology to size hedge positions. It is also relevant to the issue of validating the expectation that a hedging derivative will be effective in offsetting the risk being hedged, which is critical to qualify for special hedge accounting treatment.

We compare two regression specifications for estimating hedge ratios and measuring hedge effectiveness: a simple regression on price changes (the textbook approach) and an ECM. We analyze how these two specifications behave as the hedge horizon is extended, when the prices of the hedged item and the hedging instrument are cointegrated according to a prototypical ECM. The theoretical model under consideration imposes somewhat stringent conditions on the cash and futures price series, which are found to characterize the empirical hedging relation between gasoline cash and futures prices analyzed in our case study.

Under these conditions, we prove that the length of the hedge horizon affects the regression results in a consistent and predictable fashion. First, as the hedge horizon is lengthened, the coefficient of the error correction term from the ECM approaches negative one, whereas the hedge ratio and R^2 increase toward positive one. Second, hedgers will obtain similar results for the hedge ratio and R^2 whether they estimate an ECM, or whether they omit the error correction term and estimate a simple regression on price changes (i.e., the textbook approach).

These analytical results imply that, if the prototypical ECM characterizes the hedging relation, then it is reasonable to anticipate that the optimal hedge will be effective in offsetting the exposure, for sufficiently long hedge horizons. In addition, if adequate data are available, then hedgers can be confident in meeting the generally accepted criterion for hedge accounting (i.e., obtaining an $R^2 \geq 0.80$), if they face sufficiently long hedge horizons. Put another way, documentation of a significant, stable cointegrated hedging relation is sufficient to establish that the hedge will be “highly effective” over long hedge horizons so that, in such instances, further statistical analysis is unnecessary. Moreover, when the price series are cointegrated, it does not matter whether hedgers estimate an ECM or apply the standard textbook approach.

The finding that the hedge horizon affects the results of prescribed statistical analyses raises a number of issues for risk managers and regulators. For example, under our prototypical ECM, the hedge ratio and R^2 statistic necessarily decline as the time interval for measuring price changes shrinks. This result implies that the initial hedge ratio for a given hedging situation should be adjusted downward, as time passes and the hedge horizon diminishes. Furthermore, under current accounting guidance, the declining R^2 measure would likely force hedge accounting to be terminated prior to the natural end of the initial hedge horizon. Alternatively, if the initial hedge ratio based on the starting hedge horizon were maintained throughout the life of a hedge, the initial R^2 statistic would have little relevance to the continuing performance of the hedge as time passes throughout the initial hedge horizon.

Critically, a cointegrating relation such as the one analyzed in this study is likely to apply for many hedging situations where the pricing pertaining to the hedged item and the hedging derivative relate to a common underlying asset, (e.g., the use of gasoline futures to hedge prospective gasoline sales). Once a significant, stable cointegrating relation is established, additional hedge effectiveness tests seem to be unnecessary, as the results of these regressions are essentially pre-ordained. The situation may be different if the hedging derivative and the hedging instrument do not relate to a common underlying asset. For example, if the hedged item happens to be the price of a fabricated good, while the derivative relates to the price of a related raw material, then the respective price series may or may not have a stable cointegrating relation. In this case, the degree of hedge effectiveness may genuinely be an open question, and effectiveness testing is likely to offer useful information that is not otherwise evident.

This analytical and empirical evidence has substantive implications for risk managers and policy-makers. For example, these results suggest that regulators ought to grant special hedge accounting treatment for hedgers who can document a significant, stable cointegrating relation that conforms to our prototypical ECM. Further research should examine the impact of the hedge horizon on the hedge ratio and R^2 measure, when the cash and futures price series have a cointegrating relation that does not conform to these stringent conditions, or when the price series are not cointegrated. Additional research would also be helpful to assess the asymptotic properties of these estimation techniques, as the hedge horizon (and thus the sample size) is varied. Such analysis could help to ascertain the minimum sample size and level of statistical significance that ought to be required for these tests to justify hedge accounting treatment.

Finally, this analysis draws attention to unintended consequences that may arise when regulators implement complex rules that rely upon a specific statistical result, such as a minimum threshold for a regression R^2 , in order to

support a desired accounting rule outcome. Given the complexities that characterize all possible hedging relations which must be analyzed in practice, we suggest that the regulators consider the evidence in this study as they revisit the accounting guidance for risk management activities that pertain to special hedge accounting treatment. We hope that consideration of these issues leads to further advances toward coherent and efficacious rules that promote healthy risk management and accounting practices.

APPENDIX: PROOFS

Three lemmas are required to prove our analytical results. We first present each lemma, in turn.

Lemma 1: Suppose that Assumption 1 holds. Then:

$$\Delta_N X_t = \left[\sum_{i=0}^{N-1} \Pi(\Pi + I)^i \right] X_{t-N} + \sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i}$$

Proof: We prove this lemma by induction. For $N = 2$, we have:

$$\begin{aligned} \Delta_2 X_t &= \Delta X_t + \Delta X_{t-1} \\ &= \Pi X_{t-1} + \varepsilon_t + \Pi X_{t-2} + \varepsilon_{t-1} \\ &= \Pi[(\Pi + I)X_{t-2} + \varepsilon_{t-1}] + \varepsilon_t + \Pi X_{t-2} + \varepsilon_{t-1} \\ &= \left[\sum_{i=0}^1 \Pi(\Pi + I)^i \right] X_{t-2} + \sum_{i=0}^1 (\Pi + I)^i \varepsilon_{t-i} \end{aligned}$$

so that the statement holds for $N = 2$. Now suppose that the statement holds for $N = M$. We show that this implies that it holds for $M + 1$. Then:

$$\begin{aligned} \Delta_{M+1} X_t &= \Delta_M X_t + \Delta X_{t-M} \\ &= \left[\sum_{i=0}^{M-1} \Pi(\Pi + I)^i \right] X_{t-M} + \sum_{i=0}^{M-1} (\Pi + I)^i \varepsilon_{t-i} + \Pi X_{t-M-1} + \varepsilon_{t-M} \\ &= \left[\sum_{i=0}^{M-1} \Pi(\Pi + I)^i \right] [(\Pi + I)X_{t-M-1} + \varepsilon_{t-M}] + \sum_{i=0}^{M-1} (\Pi + I)^i \varepsilon_{t-i} + \Pi X_{t-M-1} + \varepsilon_{t-M} \\ &= \left[\sum_{i=0}^{M-1} \Pi(\Pi + I)^{i+1} + \Pi \right] X_{t-M-1} + \sum_{i=0}^{M-1} (\Pi + I)^i \varepsilon_{t-i} + \left[\sum_{i=0}^{M-1} \Pi(\Pi + I)^i + I \right] \varepsilon_{t-M} \\ &= \left[\sum_{i=0}^M \Pi(\Pi + I)^i \right] X_{t-M-1} + \sum_{i=0}^M (\Pi + I)^i \varepsilon_{t-i} \end{aligned}$$

where the last equality comes from:

$$\left[\sum_{i=0}^{M-1} \Pi(\Pi + I)^i \right] = -[I - (\Pi + I)^M],$$

so that the induction statement is true.

Lemma 2: $\begin{pmatrix} 1 + \alpha_1 & -\alpha_1 \\ 0 & 1 \end{pmatrix}^N = \begin{pmatrix} (1 + \alpha_1)^N & 1 - (1 + \alpha_1)^N \\ 0 & 1 \end{pmatrix}$

Proof: The statement is obvious for $N = 2$. Suppose that it holds for $N = M$. Then we have:

$$\begin{aligned} \begin{pmatrix} 1 + \alpha_1 & -\alpha_1 \\ 0 & 1 \end{pmatrix}^{M+1} &= \begin{pmatrix} (1 + \alpha_1)^M & 1 - (1 + \alpha_1)^M \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 + \alpha_1 & -\alpha_1 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} (1 + \alpha_1)^{M+1} & -\alpha_1(1 + \alpha_1)^M + 1 - (1 + \alpha_1)^M \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} (1 + \alpha_1)^{M+1} & 1 - (1 + \alpha_1)^{M+1} \\ 0 & 1 \end{pmatrix} \end{aligned}$$

so that Lemma 2 is true by induction.

Lemma 3: Define $v_{(N)t} = \sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i}$ and $\Omega_{(N)} = E(v_{(N)} v_{(N)}^T) = \begin{pmatrix} \sigma_{(N)1}^2 & \sigma_{(N)12} \\ \sigma_{(N)12} & \sigma_{(N)2}^2 \end{pmatrix}$,

and suppose that Assumptions 1–3 hold. Then:

$$\sigma_{(N)1}^2 = \left[\frac{1 - (1 + \alpha_1)^{2N}}{1 - (1 + \alpha_1)^2} \right] (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (2\sigma_2^2 - 2\sigma_{12}) + \sigma_2^2 N$$

$$\sigma_{(N)12} = \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12}) + \sigma_2^2 N$$

$$\sigma_{(N)2}^2 = \sigma_2^2 N$$

Proof:

$$\begin{aligned} \Omega_{(N)} &= E \left[\sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i} \right] \left[\sum_{j=0}^{N-1} \varepsilon_{t-j}^T (\Pi^T + I)^j \right] \\ &= \sum_{i=0}^{N-1} (\Pi + I)^i \Omega (\Pi^T + I)^i \\ &= \sum_{i=0}^{N-1} \begin{pmatrix} (1 + \alpha_1)^i & 1 - (1 + \alpha_1)^i \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \begin{pmatrix} (1 + \alpha_1)^i & 0 \\ 1 - (1 + \alpha_1)^i & 1 \end{pmatrix} \\ &= \sum_{i=0}^{N-1} \begin{pmatrix} (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})(1 + \alpha_1)^{2i} + 2(\sigma_{12} - \sigma_2^2)(1 + \alpha_1)^i + \sigma_2^2 & \dots \\ (\sigma_{12} - \sigma_2^2)(1 + \alpha_1)^i + \sigma_2^2 & \sigma_2^2 \end{pmatrix} \\ &= \begin{pmatrix} \left[\frac{1 - (1 + \alpha_1)^{2N}}{1 - (1 + \alpha_1)^2} \right] (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (2\sigma_2^2 - 2\sigma_{12}) + \sigma_2^2 N & \dots \\ \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12}) + \sigma_2^2 N & \sigma_2^2 N \end{pmatrix}. \end{aligned}$$

We are now ready to prove the theorems.

Theorem 1: Suppose that Assumptions 1–3 hold. Then the optimal hedge ratio for N periods ahead is given by:

$$\left[\frac{1 - (1 + \alpha_1)^N}{N\alpha_1} \right] \left(1 - \frac{\sigma_{12}}{\sigma_2^2} \right) + 1.$$

Proof of Theorem 1: Let $X_t = \begin{pmatrix} C_t \\ F_t \end{pmatrix}$. Then by Lemma 1 we have:

$$\Delta_N X_t = \left[\sum_{i=0}^{N-1} \Pi (\Pi + I)^i \right] X_{t-N} + \sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i}.$$

Define $v_{(N)t} = \sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i}$, and let

$$\Omega_{(N)} = E(v_{(N)} v_{(N)}^T) = \begin{pmatrix} \sigma_{(N)1}^2 & \sigma_{(N)12} \\ \sigma_{(N)12} & \sigma_{(N)2}^2 \end{pmatrix}.$$

Then the optimal hedge ratio is given by:

$$\begin{aligned} \frac{Cov(\Delta_N C_t, \Delta_N F_t | C_{t-N}, F_{t-N})}{Var(\Delta_N F_t | C_{t-N}, F_{t-N})} &= \frac{\sigma_{(N)12}}{\sigma_{(N)2}^2} \\ &= \frac{\left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12}) + \sigma_2^2 N}{\sigma_2^2 N} \\ &= \left[\frac{1 - (1 + \alpha_1)^N}{N\alpha_1} \right] \left(1 - \frac{\sigma_{12}}{\sigma_2^2} \right) + 1 \end{aligned}$$

Theorem 2: Suppose that Assumptions 1–4 hold. Then the population R^2 from regressing $\Delta_N C_{at}$ on $(C_{at-N} - F_{t-N})$ and $\Delta_N F_t$ is given by:

$$1 - \left(\frac{\left(\frac{c_1}{N} + \sigma_2^2 \right) - \frac{\left(\frac{c_2}{N} + \sigma_2^2 \right)^2}{\sigma_2^2}}{\frac{c_3}{N} + \sigma_2^2} \right),$$

where

$$c_1 = \left[\frac{1 - (1 + \alpha_1)^{2N}}{1 - (1 + \alpha_1)^2} \right] (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (2\sigma_2^2 - 2\sigma_{12})$$

$$c_2 = \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12})$$

$$c_3 = c_1 + \left[\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right] [1 - (1 + \alpha_1)^N]^2$$

Proof of Theorem 2: Let $w_{(N)t}$ be the residual from regressing $\Delta_N C_t$ on $(C_{t-N} - F_{t-N})$ and $\Delta_N F_t$. Then the population R^2 is given as:

$$1 - \frac{E(w_{(N)t}^2)}{E(\Delta_N C_t)^2}.$$

As ε_t is elliptically symmetric, the conditional expectation of $v_{(N)1t}$ given $\Delta_N X_{2t} = v_{(N)2t}$ is linear. Hence, we have $w_{(N)t} = v_{(N)1t} - \frac{\sigma_{(N)12}}{\sigma_{(N)2}^2} v_{(N)2t}$, so that:

$$\begin{aligned}
 E(w_{(N)t}^2) &= E\left[\left(v_{(N)1t} - \frac{\sigma_{(N)12}}{\sigma_{(N)2}^2} v_{(N)2t}\right)^2\right] \\
 &= \sigma_{(N)1}^2 + \frac{\sigma_{(N)12}^2}{\sigma_{(N)2}^4} \sigma_{(N)2}^2 - 2 \frac{\sigma_{(N)12}}{\sigma_{(N)2}^2} \sigma_{(N)12} \\
 &= \sigma_{(N)1}^2 - \frac{\sigma_{(N)12}^2}{\sigma_{(N)2}^2}
 \end{aligned}$$

The model is given as:

$$\begin{aligned}
 \Delta_N X_t &= \left[\sum_{i=0}^{N-1} \Pi(\Pi + I)^i \right] X_{t-N} + \sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i} \\
 &= \left[\sum_{i=0}^{N-1} \Pi(\Pi + I)^i \right] X_{t-N} + v_{(N)t}
 \end{aligned}$$

From Lemma 2, we have:

$$\begin{aligned}
 \sum_{i=0}^{N-1} \Pi(\Pi + I)^i &= \sum_{i=0}^{N-1} \begin{pmatrix} \alpha_1 & -\alpha_1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} (1 + \alpha_1)^i & 1 - (1 + \alpha_1)^i \\ 0 & 1 \end{pmatrix} \\
 &= \sum_{i=0}^{N-1} \begin{pmatrix} \alpha_1(1 + \alpha_1)^i & -\alpha_1(1 + \alpha_1)^i \\ 0 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} (1 + \alpha_1)^N - 1 & 1 - (1 + \alpha_1)^N \\ 0 & 0 \end{pmatrix}
 \end{aligned}$$

so that the first row of this model can be written as:

$$\Delta_{(N)} C_t = [(1 + \alpha_1)^N - 1](C_{t-N} - F_{t-N}) + v_{(N)1t}.$$

Then $E(\Delta_N C_t)^2 = [(1 + \alpha_1)^N - 1]^2 \text{Var}(C_{t-N} - F_{t-N}) + \sigma_{(N)1}^2$, as $v_{(N)t}$ is uncorrelated with $C_{t-N} - F_{t-N}$. The proof is completed by finding the final variance. Note from the model that we can write $X_t = (\Pi + I) X_{t-1} + \varepsilon_t$ so that:

$$\begin{aligned}
 \beta^T X_t &= \beta^T (\Pi + I) X_{t-1} + \beta^T \varepsilon_t \\
 &= (1 \quad -1) \begin{pmatrix} 1 + \alpha_1 & -\alpha_1 \\ 0 & 1 \end{pmatrix} X_{t-1} + \beta^T \varepsilon_t \\
 &= (1 + \alpha_1) \beta^T X_{t-1} + \beta^T \varepsilon_t
 \end{aligned}$$

where the last equality comes from Assumption 2. By Assumption 3, this is a stationary autoregressive model so that the variance of $\beta^T X_t$ is given by:

$$\frac{\text{Var}(\beta^T \varepsilon_t)}{1 - (1 + \alpha_1)} = \frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1}$$

Combining terms, the population R^2 is given by:

$$1 - \left(\frac{\sigma_{(N)1}^2 - \frac{\sigma_{(N)12}^2}{\sigma_{(N)2}^2}}{\left[\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right] [1 - (1 + \alpha_1)^N]^2 + \sigma_{(N)1}^2} \right)$$

Substituting the values for $\sigma_{(N)1}^2$, $\sigma_{(N)2}^2$, $\sigma_{(N)12}$ gives:

$$1 - \left(\frac{\left(\frac{c_1}{N} + \sigma_2^2 \right) - \frac{\left(\frac{c_2}{N} + \sigma_2^2 \right)^2}{\sigma_2^2}}{\frac{c_3}{N} + \sigma_2^2} \right),$$

with:

$$c_1 = \left[\frac{1 - (1 + \alpha_1)^{2N}}{1 - (1 + \alpha_1)^2} \right] (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (2\sigma_2^2 - 2\sigma_{12})$$

$$c_2 = \left[\frac{1 - (1 + \alpha_1)^N}{\alpha_1} \right] (\sigma_2^2 - \sigma_{12})$$

$$c_3 = c_1 + \left[\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right] [1 - (1 + \alpha_1)^N]^2$$

Corollary 2: Suppose that Assumptions 1–4 hold and $\alpha_1 \neq 0$. Then the population R^2 from regressing $\Delta_N C_{at}$ on $\Delta_N F_t$ is given by:

$$R^{*2} = R^2 + \frac{\frac{c_4}{N}}{\frac{c_3}{N} + \sigma_2^2},$$

where

$$c_4 = [(1 + \alpha_1)^N - 1]^2 \left(\frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{-\alpha_1} \right).$$

and R^2 is defined as in Theorem 2. This corollary means that, if the error correction term belongs in the model but is omitted, the regression R^2 will still approach one as N increases.

Proof of Corollary 2: Let $w_{(N)t}^*$ be the residual from regressing $\Delta_N C_t$ on only $\Delta_N F_t$. Then the population R^{*2} is given as:

$$1 - \frac{E(w_{(N)t}^{*2})}{E(\Delta_N C_t)^2}$$

As ε_t is elliptically symmetric, the conditional expectation of $v_{(N)1t}$ given $\Delta_N X_{2t} = v_{(N)2t}$ is linear. From the relationship $\Delta_N C_t = [(1 + \alpha_1)^N - 1](C_{t-N} - F_{t-N}) + v_{(N)1t}$ and the fact that the expectation of $(C_{t-N} - F_{t-N})$ does not depend on $(F_t - F_{t-N})$, we have:

$$w_{(N)t}^* = [(1 + \alpha_1)^N - 1](C_{t-N} - F_{t-N}) + v_{(N)1t} - \frac{\sigma_{(N)12}}{\sigma_{(N)2}^2} v_{(N)2t},$$

and the result follows from the same arguments in Theorem 2.

Theorem 3: Suppose that Assumptions 1, 2, and 4 hold, but $\alpha_1 = 0$. Then the population R^2 from regressing $\Delta_N C_t$ on $(C_{t-N} - F_{t-N})$ and $\Delta_N F_t$ does not depend upon N , and is always given by:

$$\frac{\sigma_{12}^2}{\sigma_1^2 \sigma_2^2}$$

Proof of Theorem 3: If $\alpha_1 = 0$, then:

$$\begin{aligned} \Omega_{(N)} &= E \left[\sum_{i=0}^{N-1} (\Pi + I)^i \varepsilon_{t-i} \right] \left[\sum_{j=0}^{N-1} \varepsilon_{t-j}^T (\Pi^T + I)^j \right] \\ &= \sum_{i=0}^{N-1} (\Pi + I)^i \Omega (\Pi^T + I)^i \\ &= \sum_{i=0}^{N-1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} N\sigma_1^2 & N\sigma_{12} \\ N\sigma_{12} & N\sigma_2^2 \end{pmatrix} \end{aligned}$$

and $\Delta_N C_t = v_{(N)1t}$ so that the population R^2 is:

$$1 - \left(\left[\frac{\sigma_{(N)1}^2 - \frac{\sigma_{(N)12}^2}{\sigma_{(N)2}^2}}{\sigma_{(N)1}^2} \right] \right) = \frac{\sigma_{12}^2}{\sigma_1^2 \sigma_2^2}$$

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