Interest Rates & Bond Portfolio Management

I. Background & Motivation.

A. Bond Portfolio managers are interest rate timers.

1. If you expect rates to decline, buy bonds.
2. If you expect rates to rise, sell bonds.

B. Before 1970's, bond valuation was considered simple & dull subject.
Since 1970's, more complex & interesting.

1. Interest Rates are more volatile.
2. Bonds are more complex:
   a. Timing of cash flows more variable.
   b. Payment of cash flows more risky (less certain).

C. For example, during the 1990's, returns from holding bonds were volatile.

1. Table 20.1 - shows 10-Yr Govt Bond Yield & Yearly Returns on Bond Portfolios.
2. Observe - when 10-Yr Govt Bond Yield ↓, Bond Portfolios ↑ (& visa versa).
   - Long Term Bonds more volatile than Medium Term Bonds.
   - Corporate Bonds (generally) pay higher rate than Government Bonds.

E&G: Table 20.1 Yields & Rates on Selected Diversified Bond Portfolios

	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Avg Yield:											
Govt 10-Yr	8.07	6.70	6.69	5.80	7.83	5.58	6.42	5.75	4.65	6.44	5.12
Yearly Returns:											
Med. Govt	9.21	13.28	6.78	7.91	-1.72	13.57	4.03	7.49	8.21	0.51	10.02
L.T. Govt	6.55	17.46	8.05	16.38	-7.69	27.47	-0.45	14.49	12.83	-8.97	18.74
Med Corp	7.42	15.49	7.98	10.65	-2.60	17.57	3.98	8.11	8.04	0.20	8.94
L.T. Corp	6.51	19.28	9.11	12.98	-5.65	25.03	2.47	12.95	8.84	-5.84	8.41

II. **Digression: Continuous Compounding & Discounting.**

A. Amount **A** invested for **n** years at **R** per annum.

1. compounded once/year, get $A(1+R)^n$.
2. compounded m times/year, get $A(1+R/m)^{mn}$.
3. Example: for n = 1 year, A=\$100, R=10%;

m=1:	get \$100 x 1.1	= \$110.
m=2:	get \$100 x 1.05^2	= \$110.25
m=4:	get \$100 x 1.025^4	= \$110.38
$m\to\infty$:	get $Ae^{Rn} = \$100e^{.10}$	= \$110.52

B. Continuous compounding, Ae^{Rn}; $e^{Rn} \approx (1+R)^n$;

Continuous discounting, Ae^{-Rn}; $e^{-Rn} \approx 1/(1+R)^n$.

C. Graph.

$$e = \lim_{m\to\infty} (1 + 1/m)^m$$

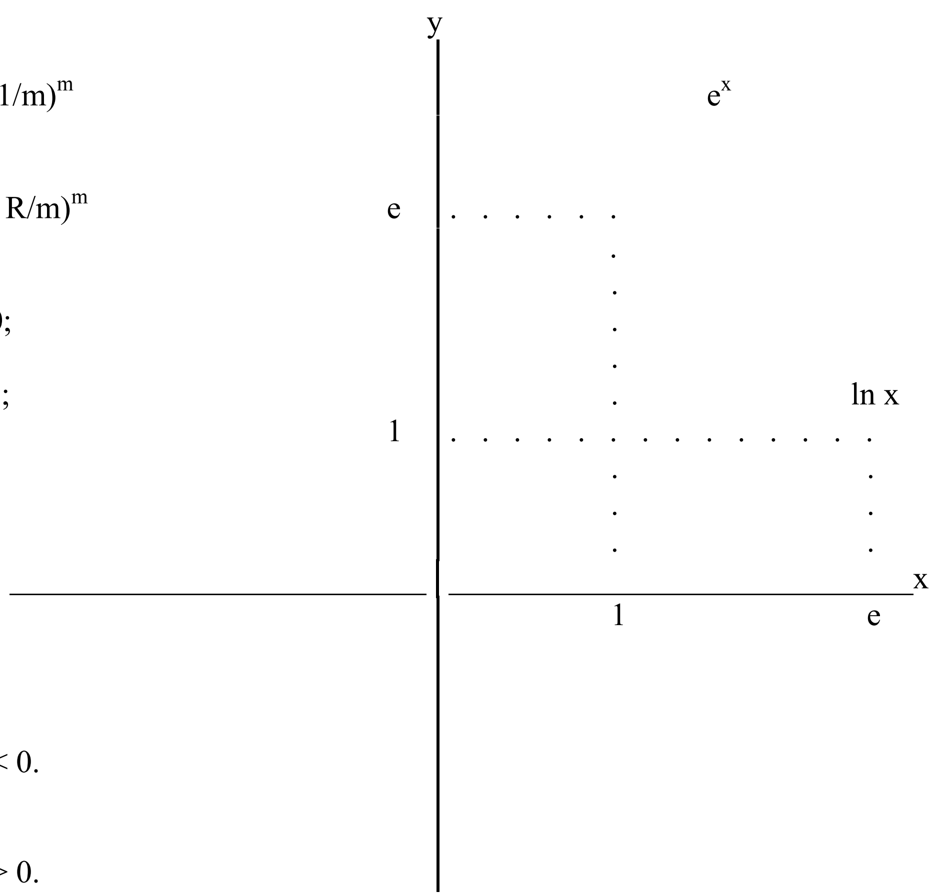
$$e^R = \lim_{m\to\infty} (1 + R/m)^m$$

Observe: $\ln(1)= 0$;

$\ln(e)= 1$;

$e^0 = 1$;

$e^1 = e$.



Discounting:
$e^x < 1$ if $x < 0$.

Compounding:
$e^x > 1$ if $x > 0$.

III. Introduction to Bonds.

A. Bonds are traded OTC, and are not very liquid (large bid-ask spread).

B. Major Categories of Bonds.

1. Federal Government Bonds – default free.
2. Corporate Bonds – risky, range from investment grade to high yield (junk).
3. Mortgages – holder has prepayment option, may pay off early if rates decline.
4. Municipal Bonds – interest payments not subject to Federal Income taxes.

C. Bond Rates.

-- How much return do investors demand in exchange for this stream of coupons?

1. spot rates (zero-coupon bond rates, used to discount future cash flows).
2. forward rates (rates on promises to borrow/lend in future).
3. **yield-to-maturity** (or bond equivalent yield, most commonly quoted rate).
   a. discount rate that equates current bond price with NPV of future coupons.
   b. most commonly cited rate in financial press.
   c. has problems – assumes all cash flows will be reinvested at this rate.
4. current yield (current year's coupon / current bond price -- not very useful).

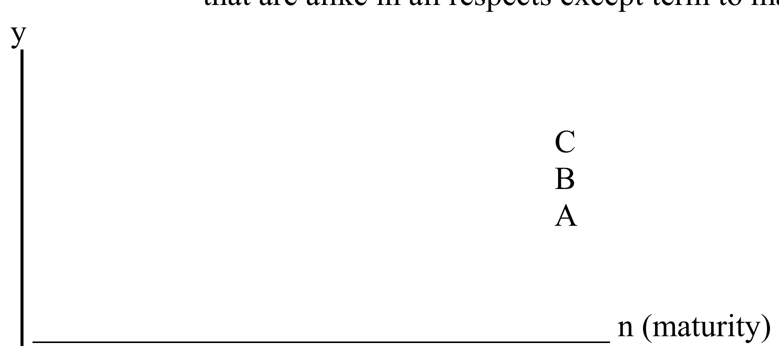
E. Bond Prices.

-- How much are investors willing to pay for this promised stream of coupons?

1. Bond promises holder payments, c_i, at future times, t_i (i = 1- n).
2. Bond Price, B, and **yield-to-maturity, y**, are related: $\mathbf{B = \sum_{i=1}^{n} c_i e^{-y\,(t_i)}}$.

D. Determinants of Bond Rates & Bond Prices.

1. Maturity – length of time before bond matures.
   a. **Yield Curve** – relation among interest rates on securities that are alike in all respects except term to maturity.



2. Default Risk – risk of not receiving future cash flows.
3. Tax status – municipals pay lower rates, since tax exempt.
4. Special Provisions (options) – callable, higher return; convertible, lower return.
5. Amount of coupon.

IV. Bond Portfolio Management – Risk of a Bond Portfolio.

A. **Duration**.

1. A measure of how long, on average, the holder of a bond has to wait before receiving cash payments.

   -- A zero-coupon bond maturing in n years has $D = n$;
   But a coupon-bearing bond maturing in n years has $D < n$,
   because some cash payments are received prior to year n.

   -- Critical concept for use in hedging bond portfolios.

2. Assume today is time t_0.
   Bond provides holder with payments, c_i, at future times, t_i ($i = 1$-n).

3. Then Bond Price, B, and yield, y, are related: $\mathbf{B = \sum_{i=1}^{n} c_i e^{-y (t_i)}}$.

4. Duration is defined as:

$$D = \left[\sum_{i=1}^{n} t_i c_i e^{-y (t_i)} \right] / B;$$

or:

$$D = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-y (t_i)}}{B} \right]$$

5. Notes:

   a. The term in brackets is NPV(payment at time t_i), divided by the bond price, B, which is the NPV(all payments).

   b. Thus, D is weighted avg of the *times when payments are made* with the weight applied to time t_i being the proportion of B provided by the payment at time t_i.

   c. The weights sum to 1.

B. Derivation of Duration formula.

1. Given the formula for the Bond Price: $B = \sum_{i=1}^{n} c_i e^{-y(ti)}$,

   we are interested in the sensitivity of bond price, B, to change in yield, y:

   $\partial B/\partial y = -\sum_{i=1}^{n} (t_i) c_i e^{-y(ti)} = -BD.$

2. Intuition:

   a. If there is a parallel shift in the yield curve, increasing all rates by Δy, this also increases all bond rates by Δy.

   b. Thus, bond prices will change by ΔB, where

$\Delta B/\Delta y = -BD$ or $\Delta B = -BD\Delta y$ or $\Delta B/B = -D\Delta y$

   c. Thus, the % change in a bond price = (-1) x (duration) x (size of parallel shift in yield curve).

** This relationship enables bond portfolio manager to assess the sensitivity of a bond's price to small changes in its yield.

C. Example calculating bond duration.

1. Consider the following bond:

   Face Value = \$100;
   10% coupon (semi-annual payments of \$5);
   n = 3 years to maturity;
   y = 12% p.a.;

Time (t_i)	Payment c_i	$PV_i(c_i)$ $c_i e^{-y\,(ti)}$	Weight $PV_i \div B$	Time x Weight $(t_i) \times W_i$
0.5	\$5	\$4.709	.050	.025
1.0	\$5	\$4.435	.047	.047
1.5	\$5	\$4.176	.044	.066
2.0	\$5	\$3.433	.042	.084
2.5	\$5	\$3.704	.039	.098
3.0	\$105	\$73.256	.778	2.334
Total	\$130	B=\$94.213	1.000	D=2.654

2. Notes:
   a. Column 3 shows PV(payments) using y=12% (e.g. PV(first payment) = $\$5e^{-.12x.5}$).
   b. Sum of Column 3 gives Bond price.
   c. Weights in Column 4 = (numbers in Column 3) ÷ B.
   d. Sum of Column 5 gives D.

3. Implications:
   a. From our formula: $\Delta B = -BD\Delta y$,

      $\Delta B = -\$94.213 \times 2.654\Delta y = -\$250.04 \times \Delta y$.

   b. Thus, if $\Delta y = +.001$, so that y increases to .121,

      expect B to change by -\$250.04 x .001 = -\$.25;

      expect B to decrease to \$94.213 - \$.25 = \$93.963

      (Can verify by recomputing column 3 with y =.121).

D. What affects Duration?

1. Table 21.2 shows what happens to duration for different bonds.

E&G Table 21.2 Duration of Bonds with different maturities & coupons

Years to Maturity:	3	5	10
Coupon			
4	2.88	4.57	7.95
6	2.82	4.41	7.42
8	2.78	4.28	7.04
10	2.74	4.17	6.76
12	2.70	4.07	6.54
14	2.66	3.99	6.36

2. This table & formula show that Duration depends on:

   a. Coupon -- higher coupon lowers D; get more cash earlier.

   b. Maturity -- longer maturity lengthens duration (almost always).

   c. Rate -- higher rate lowers D; early cash worth more, weighted more.

E. Extensions of Duration concept.

1. Duration of a *bond portfolio* can be defined as weighted avg of the durations of the individual bonds in pf, with the weights being proportional to the bond prices.

   a. $D_p = \Sigma \; w_i D_i$, where D_i = duration of bond i.

2. We have: $\Delta B/\Delta y = -BD$ or $\Delta B = -BD\Delta y$ or $\Delta B/B = -D\Delta y$.

   a. These equations also apply to a bond portfolio, as long as the yields of all bonds in the portfolio are assumed to change by the same amount.

   b. Thus, the proportional effect of a parallel shift of Δy in the yield curve is the duration of the *bond portfolio* multiplied by Δy.

3. This analysis based on assumption that y is expressed with continuous compounding.

   a. If y is expressed with annual compounding, then $\Delta B = -(BD\Delta y)/(1+y)$.

   b. If y is expressed with compounding m times per year, $\Delta B = -(BD\Delta y)/(1+y/m)$.

   c. The expression, $D/(1+y/m)$, is called *modified duration*.

   d. There are many other formulas for duration that apply to different assumptions about the slope of the yield curve, the frequency and manner of compounding, etc.

F. Limitations.

1. The concept of duration provides a simple approach to interest rate risk management.

   a. For *small, parallel shifts in yield curve*, the change in value of a pf depends solely on its duration.

   b. However, this hedging technique is far from perfect, for two reasons;
      (i) assumption of parallel shifts in yield curve;
      (ii) convexity.

2. Discussion: A portfolio of fixed income securities can be described by its avg duration.

   a. Financial institutions often try to match the avg duration of their assets with the avg duration of their liabilities.

      Called *duration matching* or *portfolio immunization*.

   b. This practice is based on assumption that yield curve always displays parallel shifts.

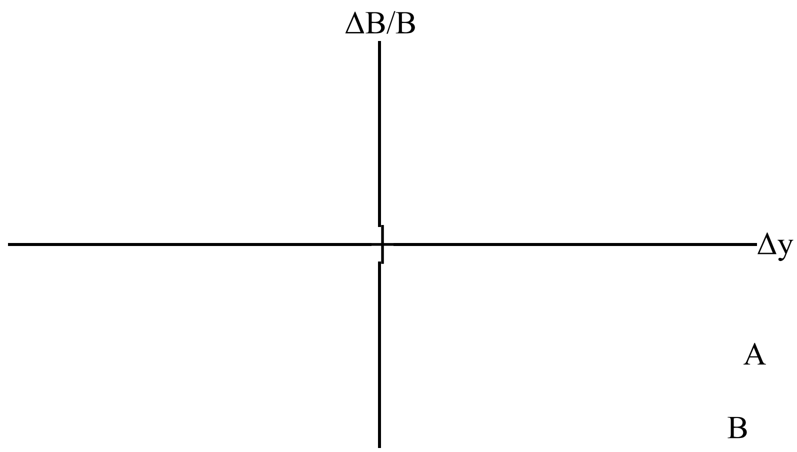
   c. When durations are matched, a small parallel shift in yield curve should have little effect on the whole pf.

      The equation, $\Delta B = -BD\Delta y$, shows that the gain (loss) on asset pf should be offset by the loss (gain) on the liability pf.

   d. However, given a ***large*** change in interest rates, *convexity* may become a problem.

G. **Convexity**.

1. Consider graph of relationship between ($\Delta B/B$) and (Δy) for two bond portfolios (A & B) having the same duration:



2. Slopes for A & B are same for current yield (at origin). Thus, for small (Δy), ($\Delta B/B$) is same for both portfolios.

3. However, for large (Δy), ($\Delta B/B$) behaves differently.

   a. Pf A has more convexity. Thus, for pf A,
      ($\Delta B/B$) ↑ by greater amount when yields ↓ ($\Delta y<0$),
      ($\Delta B/B$) ↓ by smaller amount when yields ↑ ($\Delta y>0$).

4. Note: Pf A performs better than B!

   a. For long positions in bond pfs with same duration, a high convexity pf is more attractive. (Also more expensive.)

5. Convexity of bond pf is greatest when payments are spread evenly over long period.

   Convexity of bond pf is least when payments are concentrated around a short period.

6. One measure of convexity:

$$\partial^2 B/\partial y^2 = \sum_{i=1}^{n} (t_i^2)\, c_i\, e^{-y\,(t_i)}$$

   a. Banks try to match both duration and convexity when hedging.

V. Bond Portfolio Management – Expected Return of a Bond Portfolio

A. Passive Strategies - Indexing.

1. Few active managers beat the broad bond indexes.
2. Managers wish to duplicate the indexes cheaply.
3. With stock indexes, can just buy stocks in the index.
4. Not so with bond indexes.
   a. Bonds in index are illiquid. Some bonds not available for sale at all.
5. Can buy portfolio of bonds that match the characteristics of a bond index:
   a. Maturity.
   b. Duration, convexity.
   c. Default Risk (Govt, Corp Aaa, Baa, …).
   d. Tax status (Muni's).
   e. Special Provisions (options) – call features, convertibility features.
   f. Amount of coupon.
   g. Sector of economy.
6. Bond managers can effectively replicate index returns in this way.

B. Active Strategies.

1. Can try to predict the direction of interest rates & time market moves.
   a. If expect interest rates to rise, shorten the duration of portfolio.
   b. If expect interest rates to fall, lengthen the duration of portfolio.
2. Can try to predict changes in the slope of the yield curve.
3. Can try to predict changes in credit spreads (between Govt & Aaa, Baa, …).
4. Can try to predict performance of certain sectors of bond market.
5. Can try to find miss-priced bonds.
   a. May think credit spread is too great for certain bonds.
   b. May think market is demanding to high a return for certain sectors.
   c. May believe market does not recognize value of embedded options.
      i. Call or Convertibility provision, Prepayment option, ...
6. Active strategies involve taking positions to benefit from predicted moves.
7. If manager can successfully do 1-6, will outperform.
   a. Difficult to do!
   b. Even if manager can predict accurately 60% of time,
      it is difficult to overcome transactions costs of rebalancing frequently,
      given the illiquid nature of bond markets.
   c. Again, few active managers beat the broad market indexes.

VI. Use of Derivatives to manage bond portfolio Risk & Expected Return.

A. Bond Markets are illiquid with high transactions costs.

1. Costs prohibit frequent portfolio rebalancing
required to manage Expected Return & Risk (duration, convexity).

B. Can trade interest rate derivatives.

1. **Forward & Futures** contracts on various debt instruments.
   a. Treasury Bills, Notes, & Bonds.
   b. Eurodollar Deposits.

2. **Options** on various debt instruments.
   a. Treasury Bonds.
   b. Futures on Treasury Bonds & Eurodollars.

3. Interest Rate **Swaps**.

   a. Promise to exchange interest payments on different bonds.
      i. Fixed rate for Floating rate.
      ii. Fixed rate for different Fixed rate.
      iii. Floating rate for different Floating rate.

   b. Promise to exchange bonds with different characteristics.
      i. Maturity.
      ii. Duration, convexity.
      iii. Default Risk (Govt, Corp Aaa, Baa, …).
      iv. Tax status.
      v. Special Provisions (options) – call features, convertibility features.
      vi. Amount of coupon.
      vii. Sector of economy.

4. These tools make it cheaper and easier to manage bond portfolios.

Problems:

1. What are the major categories of bonds? (see p. 3)

2. Describe several different measures of bond rates of return. (see p. 3)

3. What are the major determinants of bond rates and bond prices? (see p. 3)

4. What affects duration? (see p. 7)

5. Discuss the limitations of duration as a measure of bond portfolio risk. (see p. 9)

6. Distinguish between passive and active bond portfolio management strategies. (see p. 11)

7. A 5-year bond with a yield of 11% (continuously compounded) pays an 8% coupon at the end of each year.
   a. What is the bond's price?
   b. What is the bond's duration?
   c. Use the duration to calculate the effect on the bond's price of a 0.2% decrease in its yield.
   d. Re-calculate the bond's price on the basis of a 10.8% per annum yield and verify that the result is in agreement with your answer to c.
   e. Repeat c & d above, but calculate the effect of a 2.0% decrease in its yield.

Answer to 7.

a. The bond's price is:
   $B = 8e^{-.11} + 8e^{-.11x2} + 8e^{-.11x3} + 8e^{-.11x4} + 108e^{-.11x5} = \$86.80$

b. The bond's duration is:
   $D = (1 / 86.80)\ [1x8e^{-.11} + 2x8e^{-.11x2} + 3x8e^{-.11x3} + 4x8e^{-.11x4} + 5x108e^{-.11x5}\ ] = 4.256$ years

c. We know that: $\Delta B = -BD\Delta y$.
   Thus, the effect on B of a 0.2% decrease in y is: $\Delta B = -(\$86.80)x(4.256)x(-.002) = \$0.74$
   Thus, the new bond price is: $B = \$86.80 + \$0.74 = \$87.54$

d. With a decline of 0.2%, from a yield of 11% to a yield of 10.8%, the bond's new price is:
   $B = 8e^{-.108} + 8e^{-.108x2} + 8e^{-.108x3} + 8e^{-.108x4} + 108e^{-.108x5} = \$87.54$
   Note: This verifies that the result is in agreement with our answer to c.

e. First, the effect on B of a 2.0% decrease in y is: $\Delta B = (-\$86.80)x(4.256)x(.02) = \$7.40$
   Thus, the new bond price is: $B = \$86.80 + \$7.40 = \$94.20$

   With a 9% yield, the bond's new price is:
   $B = 8e^{-.09} + 8e^{-.09x2} + 8e^{-.09x3} + 8e^{-.09x4} + 108e^{-.09x5} = \$94.55$
   Note: This is different by $0.35, per $100 of par value of this bond,
   because of convexity given a large change in interest rates.