E&G, Ch. 16: APT

I. Background.

A. CAPM shows that, under many assumptions, equilibrium expected returns are linearly related to β_{im}, the relation between R_{ii} and a single factor, R_m. (i.e., equilibrium returns fall on a straight line.)

Contribution of CAPM - demonstrating how we can go from Single-Index Model to description of equilibrium.

B. Arbitrage Arguments support CAPM.

1. If there is an asset with expected return above line, it is over-valued:
   a. implies 2 assets exist with same risk, but different expected returns;
   b. violates LOP.
   c. then consider the following arbitrage portfolio; short the security with lower $E(R_i)$, buy the security with higher $E(R_j)$, use no wealth, has no risk, pays $E(R) > 0$.
   d. arbitrage activity forces $E(R_j)$ down onto line.

C. CAPM is restrictive.

1. The many assumptions – oversimplification.
2. Equilibrium falls on a line.
3. There is only one factor that influences R_i.

II. New Approach - Arbitrage Pricing Theory (APT).

A. Overview.

Uses arbitrage arguments to build model that
generates equilibrium security returns (& asset prices).

Based on LOP;
2 assets with identical risk cannot sell at diff. prices.

Strong assumptions behind CAPM unnecessary:
About utility theory;
Investors only consider mean & variance; …

B. APT Assumptions:

1. Homogeneous expectations;
2. LOP holds in equilibium;
3. Investors consider expected return & risk;
4. Multi-Index Model generates returns on any stock:

$$R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + \ldots + b_{ij}I_j + e_i$$

where a_i = $E(R_i)$ if all $I_j = 0$;
I_j = value of j^{th} index that affects R_i;
b_{ij} = sensitivity of $E(R_i)$ to I_j;
e_i = error with $E(e_i)=0$ & variance = σ_{ei}^2.

For the model to fully describe security returns, need:
$E(e_i\, e_j) = 0$ for all $i \neq j$;

$E[e_i\,(I_j - \bar{I}_j)] = 0$ for all stocks (i) & indexes (j).

Contribution of APT - demonstrating how we can go from Multi-Index Model to description of equilibrium.

C. Simple Derivation of APT.

1. Consider 2-Index Model:

   $R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + e_i$

2. If investor diversifies away unsystematic risk (σ_{ei}^2) then b_{i1} and b_{i2} represent the systematic risk of the diversified portfolio.

3. Investor is only concerned with $E(R_i)$, b_{i1}, and b_{i2}.

   i.e., given APT assumption that investors consider *expected return* and *risk*, they only need to consider 3 attributes of any diversified portfolio:
   $\{E(R_p),\ b_{p1},\ \text{and}\ b_{p2}\}$.

4. Consider 3 diversified pfs in equilibrium (A, B, & C).

   a. Each pf is characterized by its 3 attributes.

   b. Each set of 3 attributes represents point on a plane.

   c. 3 diversified pf's – 3 points on the plane.

   d. *This plane characterizes equilibrium in general.*

   e. *Equation of plane* can be determined from 3 points.

      i. Subst. values of $\{E(R_p), b_{p1}, b_{p2}\}$ for A, B, & C into general formula for equilibrium plane:

         $E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2}$;

      ii. Gives 3 equations in 3 unknowns; solve for λ's.

5. Expected Return & Risk measures
   for any portfolio (p) combining A, B, & C are:

   $E(R_p) = \Sigma X_i E(R_i)$; $\Sigma X_i = 1$;
   $b_{p1} = \Sigma X_i b_{i1}$;
   $b_{p2} = \Sigma X_i b_{i2}$.

6. Fact: Any portfolio combining A, B, & C
   must also lie on the same equilibrium plane.

7. Example: 3 diversified pf's in equilibrium:

Portfolio	Expected Return	Risk b_{i1}	Risk b_{i2}
A	15	1.0	.6
B	14	.5	1.0
C	10	.3	.2

   a. Apply 4 above, determine equilibrium plane:

   $E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2}$;

   $E(R_i) = 7.75 + 5 b_{i1} + 3.75 b_{i2}$.

   b. Consider another diversified portfolio:

   D = 1/3 A + 1/3 B + 1/3 C .

   Applying 5 above, get the attributes of D:

   $b_{D1} = 1/3(1.0) + 1/3(.5) + 1/3(.3) = .6$;
   $b_{D2} = 1/3(.6) + 1/3(1.0) + 1/3(.2) = .6$;
   $E(R_D) = 1/3(15) + 1/3(14) + 1/3(10) = 13$;

Or: $E(R_D) = 7.75 + 5(.6) + 3.75(.6) = 13$;
   (verifies that $E(R_D)$ lies on the plane).

c. Consider yet another diversified portfolio, E, with b_{p1} = .6 & b_{p2} = .6, but with $E(R_E)$ = 15%.

Same risk, higher E(Rp); above the plane!

Violates LOP; 2 pf's with same risk (b_{p1} & b_{p2}) cannot sell at different prices in equilibrium.

Arbitrage opportunity:

	Initial Cash Flow	End-of-Pd Cash Flow	Risk b_{p1}	Risk b_{p2}
Pf D (short)	+\$100	- \$113	- .6	- .6
Pf E (buy)	- \$100	+\$115	+.6	+.6
Arbitrage Pf	\$0	+\$2	0	0

Arbitrageurs keep buying Pf E, until Price of E ↑, and $E(R_E)$↓ onto plane.

8. Example establishes result; In equilibrium, all investments & portfolios must lie on a plane in $\{E(R_p), b_{p1}, b_{p2}\}$ space.

   If investment were above or below the plane, there is an arbitrage pf that yields $E(R_p) > 0$.

   Arbitrage would continue until all investments converged onto plane.

D. Equilibrium in the APT.

1. General equation for plane:

$$E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2}$$

   This is equilibrium model produced by APT, given the assumption that returns are generated by the 2-Index Model.

2. Interpreting the λ_i .

   a. $\lambda_i = dE(R_i)/db_{i1} = \uparrow$ in $E(R_i)$ given $\uparrow$ in b_{i1}.

   b. λ_1 & λ_2 are expected returns for bearing risks associated with I_1 & I_2 (risk premia!).

   c. Consider pf Z with $b_{i1} = b_{i2} = 0$.
   Zero-beta pf; no systematic risk wrt I_1 or I_2.
   Then $E(R_Z) = \lambda_0 = R_f$.

   d. Consider another pf, with $b_{i1} = 1$, & $b_{i2} = 0$.

   $E(R_i) = E(R_Z) + \lambda_1$;
   $\lambda_1 = E(R_i) - E(R_Z)$; (risk premium for I_1).

   Note: If $I_1 = R_m$, we have the CAPM:

   $$\lambda_1 = E(R_i) - E(R_Z) = [E(R_m) - R_f];$$

   and $E(R_i) = \lambda_0 + \lambda_1 b_{i1}$

   is $E(R_i) = R_f + [E(R_m) - R_f]\, \beta_{im}$.

   e. CAPM is special case of APT!
   - Equilibrium is on a line rather than a plane;
   - Only R_m is relevant in determining R_i .

f. Consider yet another pf, with $b_{i1} = 0$ and $b_{i2} = 1$.

$E(R_i) = E(R_Z) + \lambda_2$;
$\lambda_2 = E(R_i) - E(R_Z)$; (risk premium for I_2).

In general, λ_j = risk premium for I_j, and thus b_{ij} ;

The excess expected return required in equilibrium,
for bearing risk associated with the j^{th} factor.;

The extra expected return required because of
a secuity's sensitivity to the j^{th} factor.

E. The General APT Model.

Security returns are generated by the Multi-Index Model:

(*) $R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + \ldots + b_{iJ}I_J + e_i$

In (*), b_{ij} reflects responsiveness of R_i to factor j;
extent of j^{th} kind of risk, for security i.

In equilibrium, all investments & portfolios have
expected returns described by J-dimensional hyperplane:

(**) $E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \ldots + \lambda_J b_{iJ}$

where $\lambda_0 = E(R_Z) = R_f$;
and $\lambda_j = E(R_i) - E(R_Z)$ for security only sensitive to I_j .

In (**), $E(R_i)$ depends on the amount of each kind of risk
for security i (b_{ij}), and the price of each kind of risk (λ_j).

F. Comparison of APT with CAPM.

1. APT is more robust than CAPM.

- relies on fewer assumptions.

2. APT is very general.

- both a strength and a weakness;
- allows description of equilibrium
  in terms of *any* Multi-Index Model;
  but gives no guidance re: *which* Model!
- tells nothing about signs or size of λ's.
- thus, difficult to empirically test APT;
  & difficult to interpret any such results.

3. Problem: How to empirically test APT?

In CAPM, there is only one factor, $I_1 = R_m$;
$\lambda_1 = [E(R_m) - R_f]$ = excess return on Mkt pf.

In order to test CAPM, need data on β_{im} !
Can test CAPM by using S-I Model
to first estimate β_i for many firms, & then
test whether these β_i's are priced in Mkt.

In order to test APT, need data on the b_{ij} !
Must know how to use equation (*) to get b_{ij} .
In APT, the set of I_j's is not well-defined,
so equation (*) is not well-specified.

G. Testing the APT.

A. General 2-Step Procedure:

Step 1: Use (*) to identify & define relevant I's
and to produce estimates of the b_{ij}'s.

Step 2: Use this info on the b_{ij}'s to estimate (**).

B. Factor Analysis.

1. In Step 1, simultaneously determines which I_j's, and estimates the firm attributes, b_{ij}, for (*).
2. In Step 2, determine the λ_j .
3. Appealing to determine I_j's & b_{ij}'s simultaneously.
   but difficult to do statistically;
   and difficult to interpret the results.

C. Two Alternative Approaches.

Make assumptions about either the I_j's or the b_{ij}'s,
to get data on the b_{ij}'s for testing equation (**).

1. Use economic theory to hypothesize which I_j
   Might affect R_i in (*); Then estimate the b_{ij}'s.
   a. Chen, Roll, & Ross –
      {Economic growth, inflation,
      term structure premia & default risk premia}.
2. Specify the b_{ij}'s as a set of attributes
   (firm characteristics) that might affect $E(R_i)$.
   a. The b_{ij}'s might include the firm's
      dividend yield, market beta, size,
      book-to-mkt ratio, …

   Given data on the b_{ij}'s from 1 or 2,
   estimate the λ_j from (**), and thus test APT.

III. Uses of Multi-Index Models & APT.

A. Use in portfolio management growing rapidly.

1. Multi-Index models allow tighter control of risk:
   a. allow mgr to acct for more kinds of risk than R_m;
   b. allow investor to protect against risk besides R_m;
   c. allow investor to bet on risk other than R_m.
2. Thus, Multi-Index Models & APT aid in:
   a. passive management;
   b. active management;
   c. portfolio evaluation.

B. Assessing importance of sources of risk.

1. Can use Multi-Index Model & APT to measure:
   a. amount of different types of risk (b_{ij});
   b. the prices of different types of risk (λ_j);
   c. the contribution of different risks to $E(R_i)$.
2. Example: Consider a portfolio of *growth stocks*.
   a. Use Multi-Index Model to measure b_{ij} for this pf;
   b. b_{ij} are likely larger for this pf than for S&P 500 (high growth pf more sensitive to I_j than SP 500);
   c. Individual influences (indexes) have diff. contrib. to $E(R_p)$ for high growth pf than to S&P 500.
3. Once amount & price of different risks are measured, manager can hedge or bet on these risks in portfolio.

C. Different strategies toward portfolio management.

1. Passive Mgrs – believe mkt is efficient.
   a. can't find misspriced securities;
   b. hold portfolio that mimics some stock index.
2. Active Mgrs – believe mkt is not efficient.
   a. can find misspriced securities;
   b. make bets on some security or set of securities.

D. Use of Multi-Index Model in *Passive* Management.

1. Multi-Index Model can be used:
   a. To do better job of tracking a stock index;
   b. To design a passive pf for a particular client.

2. Can create pf of stocks that closely tracks an index.
   a. May be costly to buy all stocks in index.
      i. Larger indexes have more small stocks;
      ii. Smaller, illiquid stocks more costly to trade;
      iii. Larger indexes more costly to own always.
   b. Try to replicate index with smaller # of stocks.
   c. Can use Single-Index Model to build a fund that tracks R_m on avg, picking stocks with $\beta_{im} = 1$.
   d. Can use Multi-Index Model to build a fund that tracks index more closely, with stocks that match all major sources of risk (b_{ij}), not just R_m (β_{im}).

3. If fewer stocks are used in an index-matching pf;
   a. Pf less likely to track all common sources of risk;
   b. Multi-Index model has bigger advantage over S-I.

4. Options & Futures are traded on common indexes.
   a. Arbitrageurs look for violations of LOP –
      when option or futures price ≠ index value.
   b. Then arbitrageurs want to buy/sell the index.
   c. Arbitrageurs want to trade smaller # of stocks
      that tracks index closely, to do this more cheaply.
   d. Multi-Index Models can create such tracking pfs.

5. May want to match index, but excluding some stocks.
   a. Example: Socially Responsible Funds – don't invest in:
      tobacco stocks, stocks of South African firms, …
   b. Multi-Index Models can create such tracking pfs.

6. May want to match index, while forced to hold some stocks:
   a. Japanese funds required to hold some firms,
      to maintain business relations, …
   b. U.S. fund may wish to be tax efficient,
      holding winners to keep from paying capital gains taxes.
   c. Because these stocks may have certain sensitivities (b_{ij})
      out of line with the index being tracked,
      Multi-Index Model can be used to help get back in line.

7. Fund may wish to closely match some index,
   but also take a position regarding some type of risk.

   a. Example: Pension fund may have cash outflows
      to current recipients that will increase with inflation
      (i.e., COLA, or cost-of-living adjustments).

   b. Can use Multi-Index Model to construct a pf that has
      same sensitivities (b_{ij}) as index to all sources of risk,
      except zero-sensitivity to inflation risk.

8. APT adds additional insight to use of Multi-Index Model.

   a. APT measures the price of each kind of risk (the λ_j).

   b. e.g., APT tells investor expected cost of
      changing the exposure to inflation to zero.

   c. If pf doesn't want some risk (e.g., inflation risk),
      must give up some expected return.
      APT can help see how much extra $E(R_i)$
      is associated with each kind of risk (λ_j).

9. NOTE: Matching an index while making judgements
   about the amount of a certain kind of risk to take,
   requires that this risk be included in Multi-Index Model.

   Furthermore, the expected return (or cost) of these
   different kinds of exposure (different from an index)
   can only be determined from an APT model.

E. Use of Multi-Index Model in *Active* Management.

1. Most uses here parallel their use in Passive Mgt.

2. Multi-Index Model allows user to make bets on certain kinds of risk (I_j).

   a. Example: if you want to replicate S&P500 index,
      but you think inflation will be greater,
      can bet by increasing pf's exposure to inflation.

   b. Single-Index Model cannot do this.

   c. Inclusion of more indexes allows more such bets.
      e.g., May wish to include (take bets on):
      economic growth, value of $, business cycle, …

3. Can use APT to try to find misspriced securities.

   a. Analyst produces forecast of return for security i.

   b. APT then used, together with estimates of b_{ij},
      to calculate $E(R_i)$, given these risks.

   c. If $E(R_i)$ > analyst forecast, buy, …

   d. Analogous to use of CAPM (SML);
      With CAPM, if $E(R_i)$ above line, buy;
      With APT, if $E(R_i)$ above plane, buy, …

4. Form a tracking pf that outperforms some index.
   a. Form a pf from a subset of index being tracked, that matches all sources of risk of the index (b_{ij}).
   b. In selecting the subset of stocks for this pf, pick a group of stocks that match the index b_{ij}‘s, but that your analysts think are cheap.
   c. Called ‘research titled index funds.’
   d. Attempt to earn slightly > return than index, with slight loss in ability to track the index (because only a subset of stocks is used).

5. NOTE: The more target being tracked differs from diversified market pf, the more important is use of a Multi-Index Model to track sources of risk.

6. Alpha Funds.
   a. ‘Risk-Neutral’ strategy.
   b. Identify stocks that are ‘cheap’ or ‘expensive.’
   c. Use Multi-Index Model to form two pfs:
      One that matches index from ‘cheap stocks’;
      One that matches index from ‘expensive stocks.’
   d. Short the ‘expensive’ index fund; long the ‘cheap’ fund.
   e. Combined fund has zero risk (all b_{ij}’s cancel).
   f. If analysts are able to identify stocks well, should earn a residual return > 0.

F. Use of Multi-Index Fund & APT in Performance Evaluation. (E&G, Ch. 24)

1. If Market Risk is not the only source of risk, need Multi-Index Model & APT to account for alternative sources of risk and their impact on fund performance.

2. Consideration of Multi-Index Model & APT allow incorporation of many sources of risk into the examination of how well a fund performs.