Implementing Portfolio Theory -
Estimating the Correlation Structure of Security Returns.

I. Motivation.
Recall: \[ E(R_p) = \sum_{i=1}^{N} X_i E(R_i) \]
\[ \sigma^2(R_p) = \sigma_p^2 = \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_i \sigma_j \rho_{ij} \]

A. To Use Portfolio Analysis, need following inputs.
- \( E(R_i); \ i = 1, \ldots, N; \) (N estimates required)
- \( \sigma_i^2; \ i = 1, \ldots, N; \) (N estimates required)
- \( \rho_{ij}; \ i, j = 1, \ldots, N. \) (N(N-1)/2 estimates)

1. Need to forecast the correlation matrix.
If \( N = 150 \) stocks, \( N(N-1)/2 = 11,175! \)
B. Estimation requirements for $\rho_{ij}$ present a problem.

1. Organizational problems.
   a. Investment firms organize their analysts along industry categories; Analyst familiar with chemical stocks may not know correlation with steel stocks, …

2. Data problems.
   a. Large numbers of $\rho$’s to estimate.

C. Want to Simplify Computational Needs.

1. Problems have motivated development of models to describe & predict $\rho_{ij}$.
   a. Models fall into two categories:
      i. Index Models. - E&G, Chapter. 7
      ii. Averaging Techniques. - E&G, Chapter. 8

2. Assume covariation among stocks due to a single common influence (or index, the market), - the Single-Index Model.

3. Single-Index Model is used to estimate $\rho_{ij}$; to investigate efficient mkt tests & equilibrium tests; to learn more about how capital markets work.
II. Single-Index Models.

A. Overview.

1. Fact: When \( \text{mkt} \uparrow \), most stock prices \( \uparrow \).
   When \( \text{mkt} \downarrow \), most stock prices \( \downarrow \).

2. One reason stock prices are correlated is common response to market changes.

3. Recall, \( \sigma_p^2 = (1/N) \sigma^2 + (N-1)/N \sigma_{ij} \).

4. A measure of \( \rho_{ij} \) can be obtained by relating stock’s return to return on Market Index.

B. The Single-Index Model.

Basic Equation: \( R_i = \alpha_i + \beta_i R_m + \varepsilon_i; \quad i = 1 - N \)

By Construction: \( E(\varepsilon_i) = 0; \quad i = 1 - N. \)

By Assumption: \( E[\varepsilon_i (R_m - \bar{R}_m)] = 0; \quad i = 1 - N. \)
   \( E(\varepsilon_i \varepsilon_j) = 0; \quad i \neq j. \)

By Definition: \( \text{Var}(\varepsilon_i) = \sigma_{\varepsilon_i}^2; \quad i = 1 - N. \)
   \( \text{Var}(R_m) = E(R_m - \bar{R}_m)^2 = \sigma_m^2. \)
C. Elaborate on The Single-Index Model

1. Definitions:
   \( R_i \) = return on stock i;
   \( R_m \) = return on Market Index;
   \( \beta_i \) = coefficient of \( R_m \):
   \( \text{if } \beta_i > 1, \text{ stock more risky than mkt; } \)
   \( \text{if } \beta_i < 1, \text{ stock less risky than mkt; } \)
   -- Measures responsiveness of \( R_i \) to \( R_m \);
   -- Part of movements in \( R_i \) related to \( R_m \).
   \( \alpha_i \) = intercept:
   \( R_i \) may be higher or lower than \( \beta_i R_m \),
   depending on company characteristics;
   -- Nonrandom part of movements in \( R_i \)
   that are unique to firm i;
   -- If \( R_m = 0, \ E(R_i) = \alpha_i. \)
   \( \epsilon_i \) = random error or ‘noise’ for firm i;
   \( R_i \) may not \( = [\alpha_i + \beta_i R_m] \) at all times,
   for reasons specific to the firm.
   -- Random part of movements in \( R_i \)
   that are unique to firm i.
2. Assumptions:

\[ E(\varepsilon_i) = 0; \text{ randomness } \neq 0, \text{ but on avg washes out.} \]

\[ E[\varepsilon_i (R_m - E(R_m))] = 0; \]

firm’s randomness uncorrelated with \( R_m \);

If \( R_m \uparrow \), \( \varepsilon_i \) should not \( \uparrow \), & visa versa.

\[ E(\varepsilon_i \varepsilon_j) = 0; \text{ randomness in firm } i \text{ uncorrelated with randomness in firm } j; \]

The only reason \( R_i \) & \( R_j \) move together is \( R_m \).

\[ \text{Var}(\varepsilon_i) = \sigma_{\varepsilon_i}^2 = \text{amount of randomness about line.} \]

3. Implications:

If \( R_m \uparrow \), expect \( R_i \uparrow \) by \((\beta_i R_m)\), & \( R_j \uparrow \) by \((\beta_j R_m)\);

Thus, \( R_i \) & \( R_j \) correlated according to \( \beta_i, \beta_j, \) & \( \Delta R_m \).

However, if \( \varepsilon_i \uparrow \), do not expect \( R_m \) or \( \varepsilon_j \) to \( \uparrow \).

No effects beyond the market that account for \( \rho_{ij} \).

4. Observations:

\( \varepsilon_i \) is random variable \[ \text{distributed } N\{0, \sigma_{\varepsilon_i}^2\} \].

\( R_m \) is random variable \[ \text{distributed } N\{E(R_m), \sigma_m^2\} \].

\( R_i \) is random variable; depends on \( R_m \) and \( \varepsilon_i \! \text{!} \)

\[ E(R_i) = \alpha_i + \beta_i R_m \] \hspace{1cm} (Result 1).

\( \sigma_i^2 \) must also depend on \( \sigma_m^2 \) and \( \sigma_{\varepsilon_i}^2 \) \hspace{1cm} (Result 2).
The Single Index Model, cont. \( R_i = \alpha_i + \beta_i R_m + \varepsilon_i \);

Fact:
\[
\beta_i = \beta_{im} = \rho_{im} \left( \frac{\sigma_i}{\sigma_m} \right) = \frac{\sigma_{im}}{\sigma_m^2}.
\]
So, \( \beta_i = \rho_{im} \left( \frac{\sigma_i}{\sigma_m} \right) = \sigma_{im} / \sigma_m^2 \) and \( \rho_{im} = \beta_i \left( \frac{\sigma_m}{\sigma_i} \right) \).

D. Result 1.

The expected return on a security is:
\[
E(R_i) = E[\alpha_i + \beta_i R_m + \varepsilon_i] = E(\alpha_i) + \beta_i E(R_m) + E(\varepsilon_i) = \alpha_i + \beta_i E(R_m);
\]

Or \( E(R_i) = \alpha_i + \beta_i \bar{R}_m; \)

Or, \( \bar{R}_i = \alpha_i + \beta_i \bar{R}_m. \)
E. Result 2.

The variance of return on any security is:

$$\sigma^2(R_i) = \sigma_i^2 = E(R_i - \bar{R_i})^2 \phantom{[\neq \sigma_{ei}^2]}$$

Substituting for $R_i$ and $\bar{R}_i$:

$$\sigma_i^2 = E[(\alpha_i + \beta_i R_m + \varepsilon_i) - (\alpha_i + \beta_i \bar{R_m})]^2$$

$$= E[\beta_i (R_m - \bar{R_m}) + \varepsilon_i]^2$$

$$= \beta_i^2 E(R_m - \bar{R_m})^2 + 2\beta_i E[\varepsilon_i (R_i - \bar{R_m})] + E(\varepsilon_i^2)$$

Or,

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2$$

F. Result 3.

The covariance between any two securities is:

$$\sigma_{ij} = E[(R_i - \bar{R}_i)(R_j - \bar{R}_j)]$$

$$= E\{[\alpha_i + \beta_i R_m + \varepsilon_i] - [\alpha_i + \beta_i \bar{R_m}]\} \{[\alpha_j + \beta_j R_m + \varepsilon_j] - [\alpha_j + \beta_j \bar{R_m}]\}$$

$$= E[\beta_i (R_m - \bar{R_m}) + \varepsilon_i][\beta_j (R_m - \bar{R_m}) + \varepsilon_j]$$

$$= \beta_i \beta_j E(R_m - \bar{R_m})^2 + \beta_j E[\varepsilon_i (R_m - \bar{R_m})] +$$

$$+ \beta_i E[\varepsilon_j (R_m - \bar{R_m})] + E(\varepsilon_i \varepsilon_j)$$

Or,

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2$$
G. Implications.

If this specified relation (the Single-Index Model) represents the joint movement among securities, we can derive expressions for $E(R_i)$, $\sigma_i^2$, and $\sigma_{ij}$.

These are the input requirements for portfolio analysis.

Result 1: $E(R_i) = \alpha_i + \beta_i E(R_m) - [E&G, R_i, R_m]$

Result 2: $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\epsilon i}^2$

Result 3: $\sigma_{ij} = \beta_i \beta_j \sigma_m^2$

The individual security’s return is now described in relation to Market’s return: $\{E(R_m), \sigma_m^2, \text{ and } \beta_i\}$.

This allows great simplification in computing the expected return & variance of any portfolio: $\{E(R_p) \text{ and } \sigma_p^2\}$.
III. Expected Return and Variance of a Portfolio.

\[
E(R_p) = \sum_{i=1}^{N} X_i E(R_i) \quad \text{[Result 1]}
\]

\[
E(R_p) = \sum_{i=1}^{N} X_i \alpha_i + \sum_{i=1}^{N} X_i \beta_i E(R_m)
\]

\[
\sigma_p^2 = \sum_{i=1}^{N} X_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_{ij} \quad \text{[Results 2 & 3]}
\]

\[
\sigma_p^2 = \sum_{i=1}^{N} X_i^2 \beta_i^2 \sigma_m^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \beta_i \beta_j \sigma_m^2 + \sum_{i=1}^{N} X_i^2 \sigma_{ei}^2
\]

If we have estimates of:
1. \( \alpha_i \); \( i = 1 - N \); [or \( E(R_i) \)]
2. \( \beta_i \); \( i = 1 - N \); for each stock
3. \( \sigma_{ei}^2 \); \( i = 1 - N \); [or \( \sigma_i^2 \)]
4. \( \sigma_m^2 \);
5. \( E(R_m) \) for the Market

Then we can estimate \( E(R_p) \) and \( \sigma_p^2 \).
This requires \( 3N+2 \) estimates. \( [< N(N-1)/2!] \)
Note: Unnecessary to estimate $N(N-1)/2$ $\rho$’s directly. Only need to estimate the $\beta_i$’s showing how each security moves with respect to the Market.

A firm organized with analysts focusing on specific groups of stocks can easily expect these inputs.

Can also employ this model (to estimate $E(R_p)$ & $\sigma_p^2$) if analysts supply estimates of:

1. $E(R_i)$; $i = 1-N$; [replaces $\alpha_i$]
2. $\beta_i$; $i = 1-N$;
3. $\sigma_i^2$; $i = 1-N$; [Replaces $\sigma_{ii}^2$]
4. $\sigma_m^2$;
5. $E(R_m)$. 
IV. Characteristics of the Single-Index Model.

A. Portfolio $\beta_p$ and $\alpha_p$.

$$E(R_p) = \sum_{i=1}^{N} X_i \alpha_i + \sum_{i=1}^{N} X_i \beta_i \cdot E(R_m)$$

If we let the portfolio’s beta $= \beta_p = \sum X_i \beta_i$, and the portfolio’s alpha $= \alpha_p = \sum X_i \alpha_i$, then $E(R_p) = \alpha_p + \beta_p \cdot E(R_m)$.

Consider portfolio P to be Market portfolio (M). Then $E(R_p)$ must $= E(R_m)$. For this to be true, $\alpha_p = 0$ and $\beta_p = 1$.

$\rightarrow$ The Market’s beta is 1.

If $\beta_i > 1$, stock is more risky than Market;
If $\beta_i < 1$, stock is less risky than Market.

$\beta_i$ measures security’s risk.
B. Portfolio risk, $\sigma_p^2$.

$$\sigma_p^2 = \sum_{i=1}^{N} X_i^2 \beta_i^2 \sigma_m^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \beta_i \beta_j \sigma_m^2 + \sum_{i=1}^{N} X_i^2 \sigma_{\epsilon_i}^2$$

Observe:
In the double sum, $i \neq j$ yields cross-product terms.
If $i=j$, then the terms would be $X_i^2 \beta_i^2 \sigma_m^2$;
but these are just the terms in the first sum.

Thus, we can re-write as a single double-sum:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \beta_i \beta_j \sigma_m^2 + \sum_{i=1}^{N} X_i^2 \sigma_{\epsilon_i}^2$$

Or

$$\sigma_p^2 = \sum_{i=1}^{N} X_i \beta_i \sum_{j=1}^{N} X_j \beta_j \sigma_m^2 + \sum_{i=1}^{N} X_i^2 \sigma_{\epsilon_i}^2$$

Or

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^{N} X_i^2 \sigma_{\epsilon_i}^2$$
C. The Security’s contribution to $\sigma_p^2$.

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sum_{i=1}^{N} X_i^2 \sigma_{\varepsilon_i}^2$$

1. Consider naïve diversification, letting $X_i = 1/N$:

Then $\sigma_p^2 = \beta_p^2 \sigma_m^2 + \frac{1}{N} \sum_{i=1}^{N} \frac{1}{N} \sigma_{\varepsilon_i}^2$.

The last sum is the avg residual (firm-specific) risk for all securities in the portfolio.

2. As $N \to \infty$, second term $\to 0$, and $\sigma_p^2 \to \beta_p^2 \sigma_m^2$.

As $N \uparrow$, individual security risk is diversified away, but the risk from broad market movements remains.

3. Since $\sigma_m^2$ is the same, regardless of which stock, the factors that influence $\beta_p$ are the relevant factors that contribute to portfolio risk.

4. Since $\beta_p = \sum X_i \beta_i$, the contribution of security $i$ to $\sigma_p$ is its $\beta_i$; $\varepsilon_i$ and $\sigma_{\varepsilon_i}^2$ are irrelevant to $\sigma_p$ if we diversify.
5. Implications.

Again, the ‘total risk’ of an individual security is:

\[ \sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon i}^2. \]

↑
↑
nondiversifiable diversifiable
systematic nonsystematic

Since diversifiable risk can be eliminated cheaply by simply \( \uparrow \)ing \( N \), the Market will not pay a higher return for a stock with a higher \( \sigma_{\varepsilon i}^2 \).

What matters is the risk in a security that cannot be diversified away \( \rightarrow (\beta_i^2 \sigma_m^2) \).
Again, since \( \sigma_m^2 \) is the same for any security, i, \( \beta_i \) is the relevant measure of a security’s risk, as well as the security’s risk contribution to \( \sigma_p^2 \).
V. Estimating $\beta_i$.

To estimate $E(R_p)$ & $\sigma_p^2$ with Single-Index Model, we need forecasts of each firm’s $\beta_i$, to estimate $E(R_i)$, $\sigma_i^2$, and the $\rho_{ij}$ – the correlation matrix.

A. Estimating Historical $\beta$’s.

The Single-Index Model: 

$$ R_i = \alpha_i + \beta_i R_m + \varepsilon_i $$

Observe $\{R_{mt}, R_{it}\}$ over time; estimate $\{\alpha_i, \beta_i, \sigma_{\varepsilon_i}^2\}$.

1. Regression estimates of $\alpha_i$ and $\beta_i$:

$$ \hat{\beta}_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{\sum (R_{it} - \bar{R}_i)(R_{mt} - \bar{R}_m)}{\sum (R_{mt} - \bar{R}_m)^2} $$

$$ \hat{\alpha}_i = R_{it} - \hat{\beta}_i \bar{R}_{mt} $$

2. Note: $\beta_i$ is related to $\rho_{im}$.

$$ \rho_{im} = \frac{\sigma_{im}}{\sigma_i \sigma_m} = \frac{\sigma_{im} \sigma_m - \sigma_{im} \sigma_m}{\sigma_i \sigma_m \sigma_m \sigma_i} = \frac{\sigma_{im} \sigma_m}{\sigma_i \sigma_m} = \frac{\beta_i \sigma_m}{\sigma_i \sigma_m} $$
3. Consider $\sigma_{\varepsilon_i^2}$.

Suppose $\alpha_i = 4$ and $\beta_i = .7$;
Then, if $R_m \uparrow 1\%$, expect $R_i \uparrow .7\%$.

Some obs. will lie above the line, some below.
If obs. close to estimated line, $\sigma_{\varepsilon_i^2}$ small;
If obs. spread out around estimated line, $\sigma_{\varepsilon_i^2}$ big.

Wire through pipe analogy; $\sigma_{\varepsilon_i^2}$ = diameter of pipe.
More noise, less precision; less confidence in $\beta_i$.

$\sigma_{\varepsilon_i^2}$ is the variance of deviations from line.

Deviations $= [R_{it} - (\hat{\alpha}_i + \hat{\beta}_i R_{mt})] = e_{it}$

$\text{Var}(e_{it}) = (1/N) \sum_{t=1}^{N} [R_{it} - (\alpha_i + \beta_i R_{mt})]^2 = E(e_{it}^2)$. 
B. Accuracy of Historical $\beta$’s.

Reasonable to use historical data on $R_i$ and $R_m$, to estimate historical $\beta_i$ as predictor of future $\beta_i$.

1. Problems:
   a. True $\beta$’s nonstationary – change over time. 
      Goal is to predict future $\beta$’s – future $\rho$’s!
   b. Regression Model estimates of $\beta$ have ‘error’;
      – only estimates of true $\beta$.

2. Blume study:

   Computed historical $\beta$’s over 2 different 7-year periods.

<table>
<thead>
<tr>
<th>Number of Securities</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>individual securities</td>
<td>.60</td>
</tr>
<tr>
<td>2-stock portfolios</td>
<td>.73</td>
</tr>
<tr>
<td>4-stock portfolios</td>
<td>.84</td>
</tr>
<tr>
<td>7-</td>
<td>.88</td>
</tr>
<tr>
<td>10-</td>
<td>.92</td>
</tr>
<tr>
<td>20-</td>
<td>.97</td>
</tr>
<tr>
<td>35-</td>
<td>.97</td>
</tr>
<tr>
<td>50-</td>
<td>.98</td>
</tr>
</tbody>
</table>

   Compared $\beta$’s for each size portfolio across the 2 periods. 
   How did $\beta$’s change? What is correlation across periods?

   Found: 1) $\beta$’s on large pfs changed little over time;
           2) $\beta$’s on small pfs changed more over time.

   Thus, $\beta$’s on small pfs contain less info about future $\beta$’s.
3. Why Blume’s result?
   a. Why might estimates of historical $\beta$ change?
      i. because true $\beta$’s change;
      ii. historical $\beta$ estimates are measured with error.
   b. Confidence in historical $\beta$ from regression model depends on how much noise, $\sigma_{\epsilon i}^2$, around fitted line.
      i. The larger is $\sigma_{\epsilon i}^2$:
         - the less confidence in historical $\beta$;
         - the less predictive power historical $\beta$ will have for future $\beta$.
   c. Changes in historical $\beta$’s for individual stocks will wash each other out in a portfolio.
      i. Changes in true $\beta$’s will differ across stocks;
      ii. Errors in historical $\beta$ estimates differ across stocks;
      iii. Some $\beta$ estimates will $\uparrow$; some will $\downarrow$;
      iv. When combined into portfolio, individual changes & errors tend to cancel out.
      v. Thus should observe less change in portfolio $\beta$’s.
   d. Portfolio $\beta$’s are thus better predictors of future $\beta$’s than are individual security $\beta$’s.
C. Fundamental β’s.

Firm’s true risk reflects more than just how $R_i$ responds to $R_m$ (i.e., $\beta_i$).

1. Firm’s fundamentals matter!
Firm’s fundamentals should affect its risk ($\beta$):

(-) a. Dividend payout ratio (Div./Earnings);
(+ ) b. Asset growth;
(+ ) c. Leverage (Debt/Equity);
(-) d. Liquidity (Curr. Assets)/(Curr. Liab.);
(-) e. Asset Size;
(+ ) f. Earnings Variability;
(+ ) g. Earnings β (or Accounting β).

2. Cross-sectional Regression Model.
Consider info about firm’s fundamentals to improve predictions of future $\beta$.

$$\beta_i = a_0 + a_1 X_{1i} + a_2 X_{2i} + \ldots + a_3 X_{Ni} + \varepsilon_i$$

where $\beta_i$ = $i^{th}$ firm’s historical $\beta$;
$X$’s represent N fundamental factors that may affect risk for $i^{th}$ firm;
a_k = impact of $k^{th}$ fundamental on $\beta_i$.

3. The fitted values = “fundamental $\beta$’s.”
4. Barr Rosenberg’s Model.

   a. Combines information on historical $\beta$’s with information on firm fundamentals.

   b. Among his 101 variables in regression Model:

      i. market characteristics of stock:
         1. previous estimates of firm’s historical $\beta$;
         2. share trading volume;
         3. daily price trading range; …

      ii. measures of firm fundamentals:
         1. measures of Earnings variability;
         2. indicators of perceived success;
         3. firm size and age;
         4. measures of historical & perceived growth;
         5. measures of financial risk;
         6. other firm characteristics (listing, business…).

      iii. Industry dummy variables.
         To measure influence of industry characteristics on the firm’s fundamental $\beta$.

   c. Difficult to interpret all the $a_k$ estimates;
   Who cares? Goal is to forecast $\beta$.
   Seems to provide better forecasts than either historical $\beta$’s or fundamental data alone.

5. Alternative Approach: Forecast X’s, then $\beta$’s.
Forecast future firm fundamentals;
Plug in forecasted X’s to generate forecasts of $\beta_i$.

   a. With 101 variables, difficult task!
   b. Has been done with much simpler models.

6. Wells Fargo approach, E&G, Ch. 18.
D. The Market Model.

1. Same as Single-Index Model, except do not assume $E(\varepsilon_i \varepsilon_j) = 0$.

   Basic Equation: $R_i = \alpha_i + \beta_i R_m + \varepsilon_i; \quad i = 1 - N$

   By Construction: $E(\varepsilon_i) = 0; \quad i = 1 - N.$

   By Assumption: $E[\varepsilon_i (R_m - \bar{R}_m)] = 0; \quad i = 1 - N.$

   (NOT THIS) $E(\varepsilon_i \varepsilon_j) = 0; \quad$ for $i \neq j.$

   By Definition: $\text{Var}(\varepsilon_i) = \sigma_i^2; \quad i = 1 - N.$

   $\text{Var}(R_m) = E(R_m - \bar{R}_m)^2 = \sigma_m^2.$

2. Estimating $\beta_i$ is the same with the Single-Index Model or Market Model.

   a. Implications for portfolio risk are not same;

      Since Mkt Model does not assume $E(\varepsilon_i \varepsilon_j) = 0,$
      it does not lead to the simple expressions of portfolio risk implied by Single-Index Model.
      (See IV.B. discussion above.)

   b. Used in tests of Efficiency (E&G, Ch. 17).
VI. Summary

1. One goal is to forecast firm β’s & judge individual firm risk.

2. Another goal is to forecast correlation matrix for N stocks.
   Recall: need forecasts of $E(R_i)$, $\sigma_i^2$ and $\rho_{ij}$
   to forecast $E(R_p)$ and $\sigma_p^2$ – opportunities locus.

3. Single Index Model simplifies process:
   - simplifies immense data problem;
   - resolves organizational problem (firms org. by industry).

4. How well does S-I Model forecast correlation matrix?
   S-I Model: $\rho_{ij} = \sigma_{ij}/\sigma_i\sigma_j = (\beta_i\beta_j\sigma_m^2)/\sigma_i\sigma_j$.

   Studies compare forecasts from historical ρ’s
   with forecasts from S-I Model using β’s.

   Find β’s forecast future ρ’s better than historical ρ’s.

   -- S-I Model outperforms historical correlations!

5. Single Index Model useful.
   a. Simpler than estimating entire correlation matrix;
   b. Forecasts future ρ’s better than correlation matrix.