E&G, Ch. 1: Theory of Choice; Utility Analysis - Certainty

I. Summary:

All decision problems involve:

1) determining the alternatives available –
   the Opportunities Locus.

2) selecting criteria for choosing among alternatives –
   Maximize E(U);
   the Preference Structure.

3) the Solution.

Furthermore, individual solutions can be aggregated
to describe equilibrium conditions that prevail in mktplace.

We'll follow these steps to investigate
the problem of selecting risky assets – investing!

Before we get to risky assets,
consider the certainty case (no risk).

This will illustrate the solution to tougher problems.

II. Example.

A. Assume:
   1. Investor will receive with certainty
      $10,000 in pd 1 & $10,000 in pd 2; ($Y_1 = Y_2 = \$10,000$).
   2. Can save or borrow @ 5%.

B. Investor must decide how to allocate Y_i between C_i and S_i each period ($Y_i = C_i + S_i$).

C. Determine the Opportunity Set.

   1. Fig. 1.1: C_1 on horizontal axis; C_2 on vertical axis.

   2. Options include:

      a. Save nothing either pd.
         $C_1 = \$10,000$; $S_1 = 0$;
         $C_2 = \$10,000$; $S_2 = 0$. (pt. B)

      b. Save everything in pd 1.
         $C_1 = 0$; $S_1 = \$10,000$;
         $C_2 = Y_2 + S_1(1+r)$
         $= \$10,000 + \$10,000(1.05)$
         $= \$20,500$ (pt. A)

      c. Borrow all you can in pd 1 against pd 2 income.
         $C_2 = 0$; $C_1 = Y_1$ + borrowings against Y_2 (call this X).

         In pd 2 must pay back (1+r)X;
         Thus, $(1+r)X = \$10,000$,
         and $X = \$10,000/1.05 = \$9,524$.

         $C_1 = \$10,000 + \$9,524 = \$19,524$. (pt. C)

3. Observe, pts A, B, & C lie along a straight line.

Opportunities Locus:

All possible consumption patterns (C_1,C_2) on this line.

Above B; save in pd 1, so $C_2 > \$10{,}000$.
Below B; borrow in pd 1, so $C_2 < \$10{,}000$.

Equation of Opportunities Locus:

$C_2 = Y_2 + S_1(1+r)$ [S_1 may be < 0!]

or $C_2 = \$10{,}000 + (\$10{,}000 - C_1)*1.05$

or $C_2 = \$20{,}500 - 1.05*C_1$

Observe, slope of Opportunities Locus = -(1+r).

If C_1 decreases \$1; C_2 increases \$1*(1+r).

Each dollar saved in pd 1
can be invested and consumed in pd 2.

D. The Preference Structure – Indifference Curves.

1. Assume that investor:
   a. Maximizes E(U);
   b. Prefers more to less;
   c. E(U) displays diminishing MU(C_i).

2. Explanation:
   a. Investor's preference structure appears in set of indifference curves (Fig. 1.2).
   b. Along given indifference curve, E(U) is constant - indifferent! e.g. indifferent between A, B, & C on I_1.
   c. Prefer indiff curves further from origin. e.g. $I_1 > I_2 > I_3$, …
   d. Concave.
      - result of diminishing MU
      - to keep E(U) constant, each additional $1 decrease in C_1 requires more of an increase in C_2.
      - see I_4.

E. The Solution.

To determine optimal consumption pattern (C_1, C_2),
maximize E(U) subject to available opportunities.

Investor will move along opportunities locus
to higher indifference curves until tangency reached.

Fig. 1.3
Here, investor saves in pd 1,
where MRS = -(1+r).

Explanation:
Solution depends upon r
(slope of opportunities locus).

If r increases, slope = -(1+r) gets steeper;
If C_1 decreases by \$1, C_2 increases by \$1(1+r).

Fig. 1.4
If r increases enough, borrower switches to lender.
Investor, I1, would borrow if $r = r_1$;
would lend if $r = r_2$.

III. Determining equilibrium market interest rates.

A. Initially assume r = 5%; All investors either:
   1. lend @ 5% (above pt B);
   2. borrow @ 5% (below B);
   3. neither (at B).

B. Fig. 1.3 again; investor wants to lend $2,000 @ 5%.

   If we sum across all investors who wish to lend @ 5%,
→ total supply of credit @ 5%. One point on Supply Curve.

   If we sum across all investors who wish to borrow @ 5%,
→ total demand for credit @ 5%. One pt on Demand Curve.

C. Change r,
   so the amount investors wish to lend or borrow changes.
   Get new points on Supply and Demand Curves.

   Fig. 1.4 again; Consider one investor – likes C_2.
   @ r1, save $2,000 (pt D);
   @ r2, save $6,000 (pt E).

   Consider other investor – likes C_1.
   @ r1, borrow $6,000 (pt F);
   @ r2, borrow $2,000 (pt G).

   By varying r, trace out Supply and Demand for credit.

D. Market Equilibrium occurs where Supply = Demand.

Here, amount lenders wish to lend
= amount borrowers wish to borrow.

Markets clear.
No excess Supply or Demand; no tendency for r to change.

Thus, equilibrium interest rates depend on Supply & Demand,
which, in turn, depend on the analysis in Fig. 1.4.

Investor's income in the two periods → Opportunities Locus.

Investor's tastes → Preference Structure.

POINT: Need both components to analyze portfolio problems.
In addition, can aggregate across investors, and construct
models of equilibrium conditions in capital markets.

IV. Complications.

A. Multiple risky assets.

1. Keep certainty assumption (about Y_1, Y_2, and r),

2. But suppose there is more than one asset.
(Recall assumption, II.A.;
there was only one asset that pays a single r.)

e.g., now there are 2 assets;
Asset 1 pays 5%; Asset 2 pays 10%.

a. Opportunities Locus for Asset 1, same as before.
b. Opportunities Locus for Asset 2, steeper.

Fig. 1.4 again.

Vertical intercept for Asset 2 higher;
$\$10,000 + \$10,000*(1.10) = \$21,000$ (> \$20,500)

Slope for Asset 2 steeper (1.10 > 1.05).

3. If both assets available,
Lenders will buy Asset 2 @ 10%,
Borrowers will borrow Asset 1 @ 5%.

Effective Opportunities Locus is kinked - A'BC.

This situation is unstable;
Equilibrium cannot be reached where both assets exist.

Implies something is wrong with assumptions - either:
1) there is only one interest rate, r,
so that Opportunities Locus is not kinked;
2) there is *uncertainty*.

Since we observe many interest rates, *uncertainty* must be important!
To deal with *uncertainty* must develop more complex opportunity set.
- must consider portfolio analysis in the presence of *risk*.