Investments Background and Introduction

I. **Course Objectives** – to address the following Questions:

A. Basic questions:

Why do people and firms invest?

What investments are available?

How do you choose?

How do securities markets function?

Why are securities markets changing?

How do you keep informed?

B. Higher level questions:

What is the rate of return? How is it determined?

What is risk? How is it determined?

How do you measure and manage risk?

How are risk and return related?

How do individual securities behave?

How do portfolios of securities behave?

How do you analyze investment opportunities?

II. **Background Terminology**.

Surplus Spending Unit (SSU) - prefers to spend on current consumption and investment goods <u>less</u> than current income at current market prices.

Deficit Spending Unit (DSU) - prefers to spend on current consumption and investment goods <u>more</u> than current income at current market prices.

Investment - transaction between SSU and DSU;
SSU trades known $ amount today for expected future stream of benefits.
DSU buys known $ amount today with promise of future stream of benefits.

Security; investment vehicle; financial claim or instrument - legal contract that represents the right to receive future benefits under a stated set of conditions.

Financial Market - anywhere securities are traded; SSU's buy, DSU's sell.

Financial Institution - organization that helps SSU's & DSU's trade securities; brokerage firms, investment banks, commercial banks, S&Ls, mutual funds, ...

Financial Broker - institution that brings SSU's & DSU's together for a fee; brokerage firms, investment banks, etc.

Financial Intermediary - institution that goes between SSU's & DSU's; commercial banks, S&L's, credit unions, mutual funds, …

Organized Exchange - centralized location where organized trading is conducted in certain financial instruments under a specific set of rules (NYSE, AMEX, …).

Over-the-Counter (OTC) Market - any market where a transaction takes place other than an organized exchange; e.g. through a financial institution or broker.

Yield Curve - relation among interest rates on
securities that are alike in all respects except term to maturity.

Eurodollars - dollar-denominated deposits outside the jurisdiction of the U.S. regulators; often carry higher rates than similar U.S. instruments (LIBOR).

LIBOR - London Interbank Offer Rate; rate charged on short term Eurodollar deposits; benchmark floating rate for international borrowing/lending in $.

Financial Derivative - financial claim whose value is contingent upon movements in some underlying variable such as a stock or stock index, interest rates, exchange rates, or commodity prices;
includes forwards, futures, options, SWAPs, & combinations of these.

Margin - good faith collateral deposit, specified as a percentage of the value of the financial instrument in question; ensures integrity of market.

Arbitrage Portfolio - uses none of your own wealth, and has no risk.

III. **Basic Investment Building Blocks**.

Debt Instrument, Bond - prior, fixed claim against earnings and assets of firm.

Equity Instrument, Stock - residual ownership claim against earnings and assets.

Forward Contract - commitment by two parties to exchange a particular good for a specific price (F) at a specified future time (Expiration, Maturity);

no cash exchanges hands initially; at expiration cash is exchanged for the consideration specified in the contract;

traded OTC; customer typically required to post margin.

Futures Contract - same as Forward except that profits/losses are computed and settled (marked-to-market) on a daily basis rather than only at maturity; typically traded on organized exchange.

Financial Futures - futures contract in which the good to be delivered is a financial instrument.

Option Contract - entitles holder to *buy/sell* a certain asset *at or before* a **certain time** at a **specified price**.
Note that this gives holder the right (not the obligation) to do something.
Call - ... *buy* ...
Put - ... *sell* ...
European Option - ... *at* a certain time (not before) ...
American Option - ... *at or before* a certain time ...
Expiration/Maturity - the **certain time**.
Exercise Price/Strike Price - the **specified price**.

A Call is: in-the-money (itm) if current price > exercise price;
at-the-money (atm) if current price = exercise price;
out-of-money (otm) if current price < exercise price.

Warrant - option issued by parent corporation or financial institution, rather than by another investor, to achieve some objective.

IV. **What is the Rate of Return or Interest Rate? How is it determined?**

A. Every asset has two prices - asset or ownership price (P) & rental price (R).

1. If you don't have something & want to enjoy it, can buy or can rent.

2. For example, if you don't have money & want to spend it,
   can "buy" money with your labor, or rent money (borrow).

3. Renting money creates credit.

4. The interest rate (R) is the price of renting money – the price of credit.

5. If you rent someone else's money, you are a DSU; you pay R.

6. If someone else rents your money, you are a SSU; you earn R.

7. Interaction of DSUs & SSUs forms the market for credit.

8. The funds of SSUs determine the supply of credit $\rightarrow$ f(R).
   The needs of DSUs determine the demand for credit $\rightarrow$ f(R).

9. Together, supply & demand for credit determine the price of credit (R).
   (See C. below.)

B. Rate of Return or Interest Rate is the tradeoff of present consumption for the promise of a higher level of future consumption.

1. Nominal Interest Rate = R = total % return.
   a. Compensation to SSU for postponing consumption.
   b. Cost to DSU for advancing consumption before income.

2. Real Interest Rate = R_{real} = pure time value of money, or the change in purchasing power paid to the SSU.

3. If there is **Certainty** about future payoffs, then $R = R_{real}$.

4. **Uncertainty** complicates the transaction between SSU & DSU.

5. SSU's charge premium for uncertainty: $R = R_{real} + p_e + \pi$.
   a. p_e = expected inflation premium.
   b. π = risk premium.
   c. This is called the Fisher Equation.

6. First consider p_e .
   a. SSU really wants same purchasing power back + R_{real}.
   b. DSU willing to give same purchasing power + R_{real}.
   c. Inflation changes purchasing power!
   d. Example: let $R_{real} = 4\%$; F = 100; T.Bill (π=0).

      Consider different inflation outcomes:

	Get back	Should get
i. inflation = 0%;	$104	$104
ii. inflation = +2%;	$104	$106
iii. inflation = -2%;	$104	$102

7. Example shows: $R = R_{real} + p_e$.

8. Second consider π.
   a. If not a T. Bill, SSU also needs compensation for **uncertainty** (RISK!) about future payment by DSU, π.

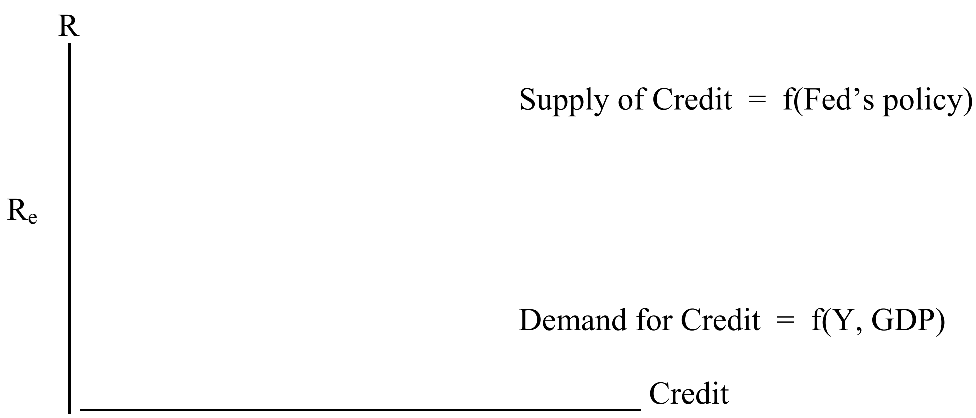
C. Determinants of Interest Rates.

1. The interest rate (R) is the price of renting money – the price of credit.
   Like all prices, interest rates are determined by demand and supply.

2. **Demand for credit** depends on the income of DSUs (Y), and national GDP.
   a. If interest rates are higher, DSUs will demand less credit
      (Demand for credit slopes downward to right).
   b. If income of DSUs ↑ (Y↑), spending ↑, demand for credit ↑ (shifts right).
   c. If economy is stronger, GDP ↑, Y↑, and demand for credit ↑ (shifts right).

3. **Supply of credit** depends on the Fed and SSU's.
   a. If interest rates are higher, SSUs will supply more credit
      (Supply of credit slopes upward to right).
   b. Fed can increase supply of credit available at banks (the money supply)
      by increasing bank reserves (shifts Supply of credit to right).

4. Supply & Demand for credit interact to determine equilibrium interest rates (R_e).

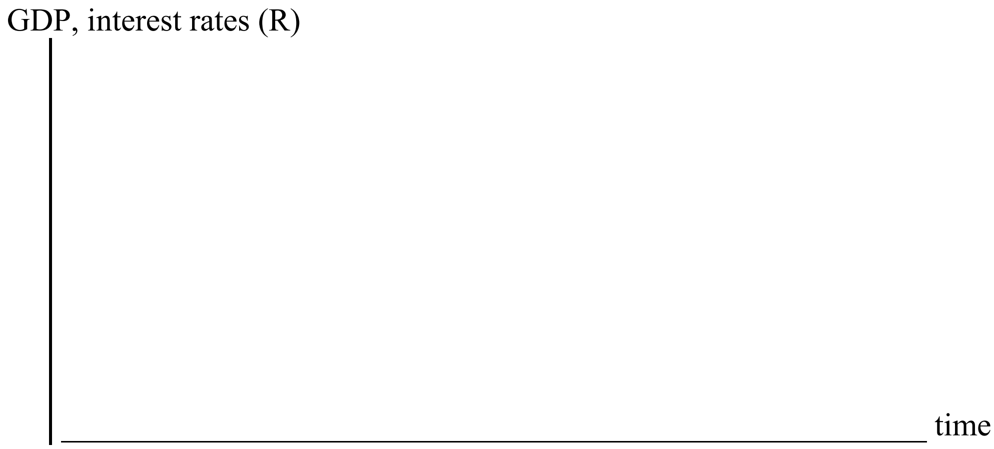


5. ***The determinants of interest rates are***
   the things that shift demand and supply for credit.
   a. If income (GDP, Y) ↑(↓), Demand for Credit shifts out (in), and R_e ↑ (↓).
   b. If the Fed ↑ (↓) reserves, Supply of Credit shifts out (in), and R_e ↓ (↑).
   c. If inflation (p_e) ↑(↓), Fisher Equation ($R = R_{real} + p_e$) implies R_e ↑ (↓).

D. Interest Rates vary with the Business Cycle.

1. As GDP ↑ (↓), Credit Demand shifts out (in), and Interest Rates ↑ (↓).
2. As GDP ↑ (↓), the Fed tightens (expands) M, and Interest Rates ↑ (↓).
3. As GDP ↑ (↓), Expected Inflation (p_e) ↑ (↓), and Interest Rates ↑ (↓).

Graphically:



E. The Yield Curve also varies with the Business Cycle.

1. When the yield curve is like A, indicates expansion & rising rates.
2. When the yield curve is like C, indicates recession & falling rates.

V. **Measures of Return and Risk**.

A. Operational definition of ex post, nominal **Return**.

R = (ending value – beginning value + cash flows) / (beginning value)

= (ending – beginning)/(beginning) + (cash flows)/(beginning)

= (% Capital Appreciation) + (% Dividend Income).

1. This is a measure of ex post, actual return earned in the past.
   a. Accounting measure of past performance.
2. For investment decisions by individuals & management,
   need to consider expected future performance!
3. To get a measure of expected future return,
   must consider possible future outcomes.
4. This requires the probability distribution function (pdf)
   that reflects your expectations of possible future outcomes.
5. Consideration of this pdf introduces uncertainty – risk.

B. What is **Risk** – uncertainty about future return from investment.

1. Reflected in investor's probability distribution function (pdf).

2. Consider the pdf's of two possible investments, I & II:

Investment:	I	II		
Possible R (R_i)	.06	.02	.06	.10
Probability (p_i)	1.0	0.2	0.6	0.2

```
        pi
1.0     |          I
        |
0.8     |                                    III:  fIII(Ri)
        |
0.6     |   II          *
        |               *
0.4     |               *                           IV:  fIV(Ri)
        |               *
0.2     |    *          *          *
        |    *          *          *
        |____________________________________________________ .  Ri
0.0         .02        .06        .10
```

3. Note: I is a T.Bill; R = .06 with certainty (no risk).
   II depends on future states of the world; uncertainty!

4. **Expected Return** = $E(R_i) = \Sigma p_i R_i$; i indexes states of world.
   a. For investment I, $E(R_i) = (1.0)(.06) = .06$
   b. For investment II, $E(R_i) = (.2)(.02) + (.6)(.06) + (.2)(.10) = .06$

5. Observe, expected return is the same for I & II, but II has more risk.

6. In reality, investor's pdf is continuous, anything between $-\infty$ & $+\infty$.
   a. Thus, pdf is more like smooth curve such as III or IV above.
   b. For continuous pdf's, expected return is defined as the integral:

$$E(R_i) = \int_{-\infty}^{\infty} f(R_i)\, R_i\, dR_i \, ; \qquad \text{analogous to } \Sigma p_i R_i .$$

C. How do we measure **Risk**? Several possibilities.

1. Range = Max$\{R_i\}$ - Min$\{R_i\}$.

   a. Problem: as sample size N increases, Range increases w/o bound.

2. Semi-Interquartile Range $= (R_{.75} - R_{.25}) / 2$
   $= (75^{th}$ %-tile - 25^{th} %-tile$) / 2$.

   a. This does not suffer from the problem with the Range.

   b. Used when the variance does not exist.

3. Variance $= \sigma^2 = E[R_i - E(R_i)]^2 = \Sigma\, p_i\, [R_i - E(R_i)]^2$.

   a. For I, $\sigma^2 = (1.0)(.06 - .06)^2 = 0$. No uncertainty, no risk.

   b. For II,
   $$\begin{aligned}\sigma^2 &= (.2)(.02\text{-}.06)^2 + (.6)(.06\text{-}.06)^2 + (.2)(.10\text{-}.06)^2 \\ &= .00032 \quad + \quad 0 \quad + \quad .00032 \\ &= .00064\end{aligned}$$

   c. Standard Deviation, σ.
   For I, $\sigma = 0$; For II, $\sigma \approx .0253$

   d. For continuous case, $\sigma^2 = \int_{-\infty}^{\infty} f(R_i)\, [R_i - E(R_i)]^2\, dR_i$.

4. If R_i deviates further from its mean, distribution is more spread out:
   $[R_i - E(R_i)]$ is larger; $[R_i - E(R_i)]^2$ is larger; σ^2 is larger.

D. **Covariance**.

1. Defn: $\sigma_{12} = Cov(R_1, R_2) = E[R_1 - E(R_1)][R_2 - E(R_2)]$

2. Operational Defn: $\sigma_{12} = 1/n \sum_{i=1}^{n} [R_{1i} - E(R_1)][R_{2i} - E(R_2)]$

3. Case i: Suppose most combinations, (R_{1i}, R_{2i}) are in 1st & 3rd quadrants.

R_2-$E(R_2)$

First quad: $[R_1 - E(R_1)]>0$ |
$[R_2 - E(R_2)]>0$ | $\boldsymbol{\sigma_{12} > 0}$
| R_1-$E(R_1)$
Third quad: $[R_1 - E(R_1)]<0$ |
$[R_2 - E(R_2)]<0$ | $\boldsymbol{\sigma_{12} > 0}$

5. Case ii: Suppose most combinations, (R_{1i}, R_{2i}) are in 2nd & 4th quadrants.

R_2-$E(R_2)$

Second quad: $[R_1 - E(R_1)]<0$ |
$[R_2 - E(R_2)]>0$ | $\boldsymbol{\sigma_{12} < 0}$
| R_1-$E(R_1)$
Fourth quad: $[R_1 - E(R_1)]>0$ |
$[R_2 - E(R_2)]<0$ | $\boldsymbol{\sigma_{12} < 0}$

6. Case iii: Suppose observations, (R_{1i}, R_{2i}) are scattered in all 4 quadrants.

R_2-$E(R_2)$

Terms in 1st & 3rd quadrant **($\sigma_{12} > 0$)**;
Terms in 2nd & 4th quadrant **($\sigma_{12} < 0$)**;

R_1-$E(R_1)$

Altogether, terms cancel out,
so that $\boldsymbol{\sigma_{12} \approx 0}$.

Point: Sign of σ_{12} shows the *nature* of relation between R_1 & R_2 .

E. **Correlation** = ρ_{12} = $\sigma_{12} / \sigma_1 \sigma_2$.

1. Note: σ_{12} may vary between $-\infty$ & $+\infty$.

   a. If R_1 and/or R_2 vary more widely (if σ_1 and/or σ_2 larger), then $[R_1 - E(R_1)]$ &/or $[R_2 - E(R_2)]$ are larger in magnitude, and σ_{12} will be larger in mag. (depending on case i, ii, or iii).

2. Thus, magnitude of σ_{12} doesn't tell us about extent of relation.

3. Correlation fixes this problem; adjusts σ_{12} for size of σ_1 and σ_2.

   a. If σ_{12} is larger (because σ_1 and/or σ_2 larger), ρ_{12} corrects for this by dividing σ_{12} by $(\sigma_1 \sigma_2)$.

4. Result: ρ_{12} varies between -1 and +1.

   a. If $\rho_{12} = +1$, R_1 & R_2 are perfectly positively related.

   b. If $\rho_{12} = -1$, R_1 & R_2 are perfectly negatively related.

   c. If $\rho_{12} = 0$, R_1 & R_2 are unrelated.

VI. **Digression #1: Continuous Compounding & Discounting.**

A. Amount **A** invested for **n** years at **R** per annum.

1. compounded once/year, get $A(1+R)^n$.
2. compounded m times/year, get $A(1+R/m)^{mn}$.
3. Example: for n = 1 year, A=\$100, R=10%;

m=1:	get \$100 x 1.1	= \$110.
m=2:	get \$100 x 1.05^2	= \$110.25
m=4:	get \$100 x 1.025^4	= \$110.38
$m\to\infty$:	get $Ae^{Rn} = \$100e^{.10}$	= \$110.52

B. Continuous compounding, Ae^{Rn}; $e^{Rn} \approx (1+R)^n$;

Continuous discounting, Ae^{-Rn}; $e^{-Rn} \approx 1/(1+R)^n$.

C. Graph.

$$e = \lim_{m\to\infty} (1 + 1/m)^m$$

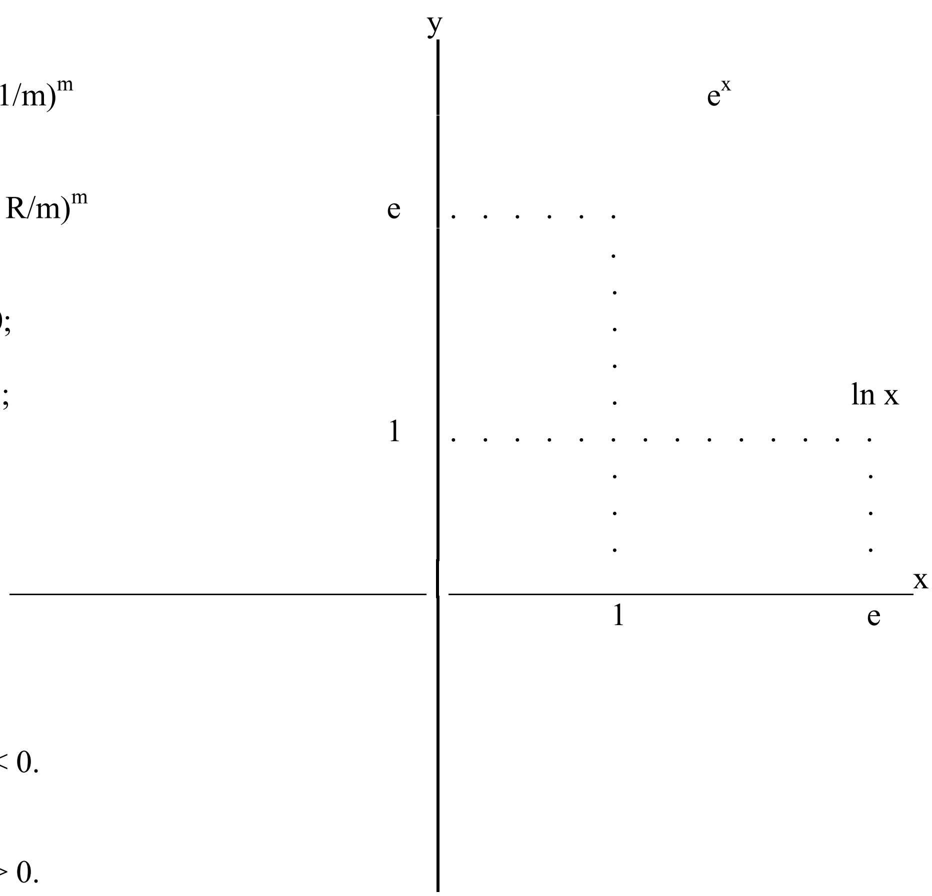
$$e^R = \lim_{m\to\infty} (1 + R/m)^m$$

Observe: $\ln(1) = 0$;

$\ln(e) = 1$;

$e^0 = 1$;

$e^1 = e$.



Discounting:
$e^x < 1$ if $x < 0$.

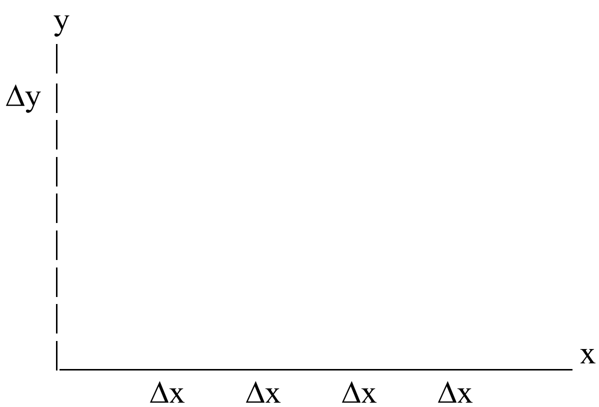
Compounding:
$e^x > 1$ if $x > 0$.

VII. **Digression #2: Optimization and Calculus**.

A. Optimization – finding max or min.

1. Slope = $\Delta y/\Delta x$ = dy/dx.

2. At the max or min, slope = 0.

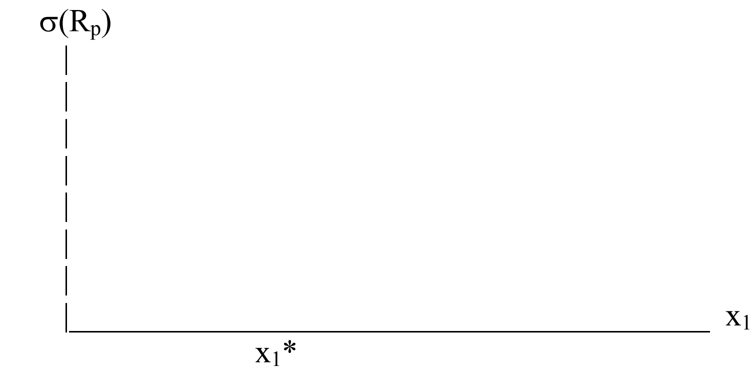


3. Can find max or min by finding the value of x where $dy/dx = 0$.

4. Procedure:
   a. Find dy/dx.
   b. Set $dy/dx = 0$.
   c. Solve for value of x where $dy/dx = 0$.

B. Use of calculus.

1. Finding portfolio that minimizes risk.

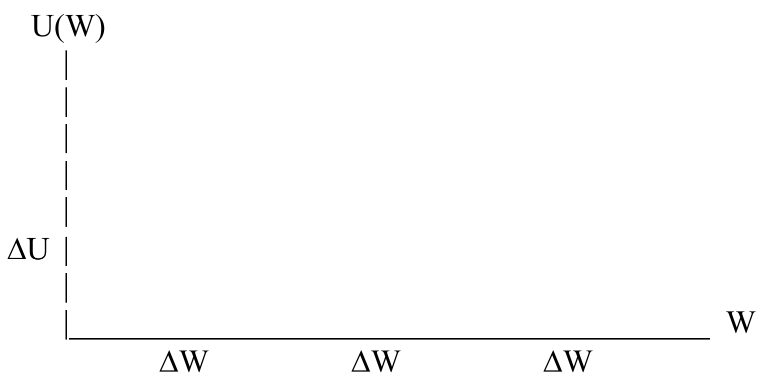


a. x_1 = portion of portfolio invested in asset 1.

b. Risk of portfolio = $\sigma(R_p)$;
depends on how much you put in asset 1.

c. Want to find portfolio with minimum risk (at x_1*).

2. Utility Function: U(W) = some function of wealth, W.



a. slope = $\Delta U/\Delta W = dU/dW = MU > 0$.

b. Concave downward; as W↑, MU↓
(each additional $ adds less incremental happiness);
diminishing MU of Wealth – **risk averse behavior**.