Elton, Gruber, Brown, and Goetzmann
Modern Portfolio Theory and Investment Analysis, 7th Edition
Solutions to Text Problems: Chapter 13

Chapter 13: Problem 1

The equation for the security market line is:

$$\overline{R_i} = R_F + \left(\overline{R_m} - R_F\right)\beta_i$$

Thus, from the data in the problem we have:

$$6 = R_F + \left(\overline{R_m} - R_F\right) \times 0.5 \text{ for asset 1}$$

$$12 = R_F + \left(\overline{R_m} - R_F\right) \times 1.5 \text{ for asset 2}$$

Solving the above two equations simultaneously, we find R_F = 3% and $\overline{R_m}$ = 9%. Using those values, an asset with a beta of 2 would have an expected return of:

$$3 + (9 - 3) \times 2 = 15\%$$

Chapter 13: Problem 2

Given the security market line in this problem, for the two stocks to be fairly priced their expected returns must be:

$$\overline{R_X} = 0.04 + 0.08 \times 0.5 = 0.08 \; (8\%)$$

$$\overline{R_Y} = 0.04 + 0.08 \times 2 = 0.20 \; (20\%)$$

If the expected return on either stock is higher than its return given above, the stock is a good buy.

Chapter 13: Problem 3

Given the security market line in this problem, the two funds' expected returns would be:

$$\overline{R_A} = 0.06 + 0.19 \times 0.8 = 0.212 \; (21.2\%)$$

$$\overline{R_B} = 0.06 + 0.19 \times 1.2 = 0.288 \; (28.8\%)$$

Comparing the above returns to the funds' actual returns, we see that both funds performed poorly, since their actual returns were below those expected given their beta risk.

Chapter 13: Problem 4

Given the security market line in this problem, the riskless rate equals 0.04 (4%), the intercept of the line, and the excess return of the market above the riskless rate (also called the "market risk premium") equals 0.10 (10%), the slope of the line. (The return on the market portfolio must therefore be 0.04 + 0.10 = 0.14, or 14%.)

Chapter 13: Problem 5

The price form of the CAPM's security market line equation is:

$$P_i = \frac{1}{r_F}\left[\overline{Y_i} - \left(\overline{Y_m} - r_F \times P_m\right) \times \frac{\operatorname{cov}(Y_i Y_m)}{\operatorname{var}(Y_m)}\right]$$

where $r_F = (1 + R_F)$ and $\overline{R_m} = \frac{\overline{Y_m} - P_m}{P_m}$.

From Problem 4, we have $R_F = 0.04$ and $\overline{R_m} = 0.14$. Therefore $0.14 = \frac{\overline{Y_m} - P_m}{P_m}$ which gives $1.14P_m = \overline{Y_m}$.

Substituting these vales into the above security market line equation, we have:

$$\begin{aligned} P_i &= \frac{1}{1.04}\left[\overline{Y_i} - \left(1.14 \times P_m - 1.04 \times P_m\right) \times \frac{\operatorname{cov}(Y_i Y_m)}{\operatorname{var}(Y_m)}\right] \\ &= \frac{1}{1.04}\left[\overline{Y_i} - 0.10 \times P_m \times \frac{\operatorname{cov}(Y_i Y_m)}{\operatorname{var}(Y_m)}\right] \end{aligned}$$

Chapter 13: Problem 6

To be rigorous, one should use the four Kuhn-Tucker conditions shown in Appendix E of Chapter 6. To find the optimum portfolio when short sales are not allowed, we have, for each asset *i*, the following Kuhn-Tucker conditions:

$$\frac{d\theta}{dX_i} + U_i = 0 \qquad (1)$$

$$X_i U_i = 0 \qquad (2)$$

$$X_i \geq 0 \qquad (3)$$

$$U_i \geq 0 \qquad (4)$$

We have already seen that, given the assumptions of the standard CAPM, setting $\frac{d\theta}{dX_i} = 0$ gives the equilibrium first order condition for asset *i*, which is the standard CAPM's security market line:

$$\overline{R_i} = R_F + \left(\overline{R_m} - R_F\right)\beta_i$$

or equivalently

$$\overline{R_i} - R_F - \left(\overline{R_m} - R_F\right)\beta_i = 0$$

When short sales are not allowed, Kuhn-Tucker condition (1) implies that:

$$\overline{R_i} - R_F - \left(\overline{R_m} - R_F\right)\beta_i + U_i = 0$$

But, since all assets are held long in the market portfolio, $X_i > 0$ for each asset and therefore, given Kuhn-Tucker condition (2), $U_i = 0$ for each asset. Thus, the standard CAPM holds even if short sales are not allowed.

Chapter 13: Problem 7

Using the two assets in Problem 1, a portfolio with a beta of 1.2 can be constructed as follows:

$$0.5X_1 + (1.5)(1 - X_1) = 1.2$$

$$X_1 = 0.3;\ X_2 = 0.7$$

The return on this combination would be:

$$0.3(6\%) + 0.7(12\%) = 10.2\%$$

Asset 3 has a higher expected return than the portfolio of assets 1 and 2, even though asset 1 and the portfolio have the same beta. Thus, buying asset 3 and financing it by shorting the portfolio would produce a positive (arbitrage) return of 15% – 10.2% = 4.8% with zero net investment and zero beta risk.

Chapter 13: Problem 8

The security market line is:

$$\overline{R_i} = R_F + \left(\overline{R_m} - R_F\right)\beta_i$$

Substituting the given values for assets 1 and 2 gives two equations with two unknowns:

$$9.4 = R_F + \left(\overline{R_m} - R_F\right) \times 0.8$$

$$13.4 = R_F + \left(\overline{R_m} - R_F\right) \times 1.3$$

Solving simultaneously gives:

$$R_F = 3\%\ ;\ \overline{R_m} = 11\%$$

Chapter 13: Problem 9

Substituting the given betas in the given equation yields:

$$\overline{R_1} = 0.178\ (17.8\%);\ \overline{R_2} = 0.151\ (15.1\%)$$