Elton, Gruber, Brown, and Goetzmann
Modern Portfolio Theory and Investment Analysis, 7th Edition
Solutions to Text Problems: Chapter 12

Chapter 12: Problem 1

Equation (12.1) in the text can be used to answer this question:

$$\frac{\overline{R_N} - R_F}{\sigma_N} > \frac{\overline{R_{US}} - R_F}{\sigma_{US}} \times \rho_{N,US}$$

As is explained in the text, if the above inequality holds, then the foreign investment will be attractive to a U.S. investor. $\overline{R_{US}}$ and $\overline{R_N}$ for the foreign countries are given in the problem's table. From the tables in the text, we have:

	σ_N	$\rho_{N,US}$
Austria	24.50	0.281
France	17.76	0.534
Japan	25.70	0.348
U.K.	15.59	0.646

Also, from the text tables, σ_{US} = 13.59. Given that R_F = 6%, we have:

	$\frac{\overline{R_N} - R_F}{\sigma_N}$	$\frac{\overline{R_{US}} - R_F}{\sigma_{US}} \times \rho_{N,US}$
Austria	0.327	0.289
France	0.563	0.550
Japan	0.311	0.358
U.K.	0.577	0.665

For Austria and France, the above inequality holds, so a U.S. investor should consider those foreign markets as attractive investments; for Japan and the U.K., the above inequality does not hold, so a U.S. investor should not consider those foreign markets as attractive investments.

Chapter 12: Problem 2

To answer this question, use the formula introduced in Chapter 5 for finding the minimum-risk portfolio of two assets:

$$X_1^{GMV} = \frac{\sigma_2^2 - \sigma_1\sigma_2\rho_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{12}}$$

where X_1 is the investment weight for asset 1 and $X_2 = 1 - X_1$.

For equities, σ_{US} = 13.59, σ_N = 16.70 and $\rho_{N,US}$ = 0.423. So the minimum-risk portfolio is:

$$\begin{aligned} X_{US}^{GMV} &= \frac{(16.7)^2 - (13.59)(16.7)(0.423)}{(13.59)^2 + (16.7)^2 - (2)(13.59)(16.7)(0.423)} \\ &= 0.6734 \ (67.34\%) \end{aligned}$$

$$X_N^{GMV} = 1 - X_{US}^{GMV} = 0.3266 \ (32.66\%)$$

For bonds, σ_{US} = 7.90, σ_N = 9.45 and $\rho_{N,US}$ = 0.527. So the minimum-risk portfolio is:

$$\begin{aligned} X_{US}^{GMV} &= \frac{(9.45)^2 - (7.9)(9.45)(0.527)}{(7.9)^2 + (9.45)^2 - (2)(7.9)(9.45)(0.527)} \\ &= 0.6841 \ (68.41\%) \end{aligned}$$

$$X_N^{GMV} = 1 - X_{US}^{GMV} = 0.3159 \ (31.59\%)$$

For T-bills, σ_{US} = 0.35, σ_N = 6.77 and $\rho_{N,US}$ = –0.220. So the minimum-risk portfolio is:

$$\begin{aligned} X_{US}^{GMV} &= \frac{(6.77)^2 - (0.35)(6.77)(-0.22)}{(0.35)^2 + (6.77)^2 - (2)(0.35)(6.77)(-0.22)} \\ &= 0.9863 \ (98.63\%) \end{aligned}$$

$$X_N^{GMV} = 1 - X_{US}^{GMV} = 0.0137 \ (1.37\%)$$

Chapter 12: Problem 3

In the text, the return due to exchange-rate changes (R_X) is shown to be equal to $fx_t/fx_{t-1} - 1$, where fx_t is the foreign exchange rate at time t expressed in terms of the investor's home currency per unit of foreign currency. Let fx_t be the exchange rate expressed in terms of dollars and fx^*_t be the exchange rate expressed in terms of pounds. These two rates are simply reciprocals, i.e., $fx^*_t = 1/fx_t$. So from the table in the problem we have:

Period	$(1 + R_X)$ (for US investor)	$(1 + R^*_X)$ (for UK investor)
1	2.5/3 = 0.833	3/2.5 = 1.200
2	2.5/2.5 = 1.000	2.5/2.5 = 1.000
3	2/2.5 = 0.800	2.5/2 = 1.250
4	1.5/2 = 0.750	2/1.5 = 1.333
5	2.5/1.5 = 1.667	1.5/2.5 = 0.600

The total return to a U.S. investor from a U.K. investment is $(1 + R_X)(1 + R_{UK}) - 1$; the total return to a U.K. investor from a U.S. investment is $(1 + R^*_X)(1 + R_{US}) - 1$. So:

Return to U.S. Investor

Period	From U.S. Investment	From U.K. Investment			
1	10%	(0.833)(1.05)	– 1	=	–12.5%
2	15%	(1)(0.95)	– 1	=	– 5.0%
3	–5%	(0.8)(1.15)	– 1	=	– 8.0%
4	12%	(0.75)(1.08)	– 1	=	–19.0%
5	6%	(1.667)(1.1)	– 1	=	83.3%
Average	7.6%				7.76%

Return to U.K. Investor

Period	From U.K. Investment	From U.S. Investment			
1	5%	(1.2)(1.1)	– 1	=	32.0%
2	–5%	(1)(1.15)	– 1	=	15.0%
3	15%	(1.25)(0.95)	– 1	=	18.75%
4	8%	(1.333)(1.12)	– 1	=	49.3%
5	10%	(0.6)(1.06)	– 1	=	–36.4%
Average	6.6%				15.73%

Chapter 12: Problem 4

Using the data and averages from Problem 3 we have:

For U.S. Investor

$$\sigma_{US} = \sqrt{\frac{(10-7.6)^2 + (15-7.6)^2 + (-5-7.6)^2 + (12-7.6)^2 + (6-7.6)^2}{5}}$$
$$= 6.95\%$$

$$\sigma_{UK} = \sqrt{\frac{(-12.5-7.76)^2 + (-5-7.76)^2 + (-8-7.76)^2 + (-19-7.76)^2 + (83.3-7.76)^2}{5}}$$
$$= 38.06\%$$

For U.K. Investor

$$\sigma_{UK} = \sqrt{\frac{(5-6.6)^2 + (-5-6.6)^2 + (15-6.6)^2 + (8-6.6)^2 + (10-6.6)^2}{5}}$$
$$= 6.65\%$$

$$\sigma_{US} = \sqrt{\frac{(32-15.73)^2 + (15-15.73)^2 + (18.75-15.73)^2 + (49.3-15.73)^2 + (-36.4-15.73)^2}{5}}$$
$$= 28.70\%$$

Chapter 12: Problem 5

This problem is essentially the same as Problem 3, except that the exchange rate is given in indirect (yen/$) terms rather than direct ($/yen) terms. From the table in the problem we have:

Period	$(1 + R_X)$ (for US investor)	$(1 + R^*_X)$ (for Japanese investor)
1	200/180 = 1.111	180/200 = 0.900
2	180/190 = 0.947	190/180 = 1.056
3	190/150 = 1.267	150/190 = 0.789
4	150/170 = 0.882	170/150 = 1.133
5	170/180 = 0.944	180/170 = 1.059

The total return to a U.S. investor from a Japan investment is $(1 + R_X)(1 + R_J) - 1$; the total return to a Japanese investor from a U.S. investment is $(1 + R^*_X)(1 + R_{US}) - 1$. So:

Return to U.S. Investor

Period	From U.S. Investment	From Japan Investment			
1	12%	(1.111)(1.18)	– 1	=	31.10%
2	15%	(0.947)(1.12)	– 1	=	6.06%
3	5%	(1.267)(1.1)	– 1	=	39.37%
4	10%	(0.882)(1.12)	– 1	=	–1.22%
5	6%	(0.944)(1.07)	– 1	=	1.01%
Average	9.6%				15.26%

Return to Japanese Investor

Period	From Japan Investment	From U.S. Investment			
1	18%	(0.9)(1.12)	– 1	=	0.80%
2	12%	(1.056)(1.15)	– 1	=	21.44%
3	10%	(0.789)(1.05)	– 1	=	–17.16%
4	12%	(1.133)(1.1)	– 1	=	24.63%
5	7%	(1.059)(1.06)	– 1	=	12.25%
Average	11.8%				8.39%

Chapter 12: Problem 6

The answers to this problem are found in the same way as those to Problem 4.

For the U.S. investor: σ_{US} = 3.72%; σ_J = 16.68%

For the Japanese investor: σ_J = 3.6%; σ_{US} = 15.227%

Chapter 12: Problem 7

Use the formula for the sample correlation coefficient ρ with five observations:

$$\rho = \frac{\sum_{t=1}^{5}\left(R_{US_t} - \overline{R_{US}}\right)\left(R_{J_t} - \overline{R_J}\right)}{\sqrt{\sum_{t=1}^{5}\left(R_{US_t} - \overline{R_{US}}\right)^2 \sum_{t=1}^{5}\left(R_{J_t} - \overline{R_J}\right)^2}}$$

For the U.S. investor, $\rho = -0.251$.

For the Japanese investor, $\rho = -0.050$.